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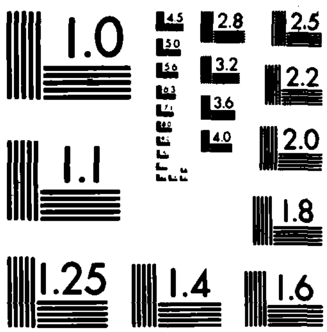

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INFORMATION LOSS CAUSED BY NOISE IN MODELS FOR DICHOTOMOUS ITEMS

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Because of the recent popularity of the three-parameter logistic model among the researchers who apply latent trait theory, it will be worthwhile to investigate the effect of noise accomodated in different models. In the present paper, four types of models on the dichotomous response level, Types A, B, C and D, are considered. Type A does not include noise, and the other three types do. Observations are made as to how much total item information is lost because of the noise, how the item response information functions are affected, how the speed of convergence to the normality of the conditional distribution of the maximum likelihood estimate of ability, given a specific ability level, is affected, and so forth.

INFORMATION LOSS CAUSED BY NOISE IN MODELS FOR DICHOTOMOUS ITEMS

ABSTRACT

Because of the recent popularity of the three-parameter logistic model among the researchers who apply latent trait theory, it will be worthwhile to investigate the effect of noise accomodated in different models. In ~~the present~~ this paper, four types of models on the dichotomous response level, Types A, B, C and D, are considered. Type A does not include noise, and the other three types do. Observations are made as to how much total item information is lost because of the noise, how the item response information functions are affected, how the speed of convergence to the normality of the conditional distribution of the maximum likelihood estimate of ability, given a specific ability level, is affected, and so forth.

A

The research was conducted at the principal investigator's laboratory, 405 Austin Peay Hall, Department of Psychology, University of Tennessee, Knoxville, Tennessee. Those who worked for her as assistants include Paul S. Changas, Charles T. McCarter, Christina C. Grey, Shiao-Yung Chen and Philip S. Livingston.

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I Introduction

Throughout this paper, we shall only consider the unidimensional latent space in the context of latent trait theory, and the dichotomous response level (Samejima, 1972), on which the item is scored either right or wrong. Let θ be the latent trait, and u_g be the binary item score for item g . The operating characteristic of the item score $u_g = 1$, or the conditional probability with which the examinee answers item g correctly, given θ , is called the item characteristic function (Lord and Novick, 1968), and is denoted by $P_g(\theta)$. This item characteristic function can be any function of θ , which relates the binary item to ability θ . When the item has a relatively simple positive relationship with ability θ , however, it is natural to consider a strictly increasing function for its item characteristic function. Many existing models follow this rule, including the normal ogive model, the logistic model, Rasch model, and the three-parameter normal ogive and logistic models. Throughout this paper, we shall solely consider strictly increasing item characteristic functions in a broader sense.

Let $A_{u_g}(\theta)$ be the basic function (Samejima, 1969, 1972) of item score u_g . We can write

$$(1.1) \qquad A_{u_g}(\theta) \begin{cases} = \dfrac{\partial}{\partial\theta} \log Q_g(\theta) & u_g = 0 \\ = \dfrac{\partial}{\partial\theta} \log P_g(\theta) & u_g = 1 \,, \end{cases}$$

where

$$(1.2) \qquad Q_g(\theta) = 1 - P_g(\theta) \,.$$

The response pattern, V, for the set of n binary items is given by

$$(1.3) \qquad V = (u_1, u_2, \ldots, u_g, \ldots, u_n)' .$$

When local independence holds, the operating characteristic, $P_V(\theta)$, of the response pattern V can be written as the product of the n operating characteristics of the item responses in V, that is

$$(1.4) \qquad P_V(\theta) = \prod_{u_g \epsilon V} P_g(\theta)^{u_g} Q_g(\theta)^{1-u_g} .$$

Since $P_V(\theta)$ is also the likelihood function, $L_V(\theta)$, used in estimating the latent trait for the examinee with the particular response pattern V, we can write for the likelihood equation

$$(1.5) \qquad \frac{\partial}{\partial\theta} \log L_V(\theta) = \frac{\partial}{\partial\theta} \log P_V(\theta) = \sum_{u_g \epsilon V} A_{u_g}(\theta) = 0 ,$$

where $A_{u_g}(\theta)$ is the basic function defined by (1.1). When a simple sufficient statistic for the response pattern V does not exist, these basic functions play an important role in obtaining the maximum likelihood estimate of ability θ, as is obvious from (1.5).

The item response information function, $I_{u_g}(\theta)$, is defined by

$$(1.6) \qquad I_{u_g}(\theta) = - \frac{\partial}{\partial\theta} A_{u_g}(\theta) .$$

Thus, if the basic function is strictly decreasing in θ, then the

item response information function is positive except, at most, at an enumerable points of θ . The item information function, $I_g(\theta)$, of item g is given as the conditional expectation of the item response information function, given θ . We can write

$$(1.7) \qquad I_g(\theta) = E[I_{u_g}(\theta)|\theta] = [\frac{\partial}{\partial\theta} Q_g(\theta)]^2 Q_g(\theta)^{-1} + [\frac{\partial}{\partial\theta} P_g(\theta)]^2 P_g(\theta)^{-1}$$

$$= [\frac{\partial}{\partial\theta} P_g(\theta)]^2 P_g(\theta)^{-1} Q_g(\theta)^{-1} .$$

It is obvious from (1.7) that the item information function is non-negative in nature. The response pattern information function, $I_V(\theta)$, is defined by

$$(1.8) \qquad I_V(\theta) = -\frac{\partial^2}{\partial\theta^2} \log P_V(\theta) = \sum_{u_g \varepsilon V} I_{u_g}(\theta) ,$$

and the test information function, $I(\theta)$, is given as the conditional expectation of the response pattern information function, given θ . Thus we can write

$$(1.9) \qquad I(\theta) = E[I_V(\theta)|\theta] = \sum_V I_V(\theta) P_V(\theta) = \sum_{g=1}^{n} I_g(\theta) .$$

When the item score is based upon the true understanding of the question, or "knowledge", by the examinee, the item characteristic function specifies the probability assigned to the strict "knowledge", given ability θ . On the other hand, if the item has a multiple-choice

format, the item score is based not only upon the "knowledge" but also upon the chance success. Thus in such a case the effect of noise more or less contaminates the resultant item characteristic function, and may cause serious problems, as was observed in the three-parameter models (Samejima, 1972, 1973b). It is advisable to avoid such contamination by devising the instructions of the test in such a way as to discourage examinees from guessing, by including informative distractors among the alternatives of the multiple-choice test item, and so forth (cf. Samejima, RR-79-4).

Because of the increasing interest in the three-parameter logistic model (Birnbaum, 1968) in practical applications, however, it will be meaningful to further investigate the effect of noise in ability estimation. In the present paper, such an attempt is made with regard to the information loss and the speed of convergence of the conditional distribution of the maximum likelihood estimate of ability θ, given θ, to normality.

II Four Types of Models for Dichotomous Test Items

We assume that the item characteristic function, $P_g(\theta)$, is strictly increasing in θ, for the interval

(2.1) $$\underline{\theta} < \theta < \bar{\theta} .$$

These lower and upper endpoints of θ can either be negative and positive infinities, respectively, or finite numbers. The interval, $(\underline{\theta}, \bar{\theta})$, can either be the whole range of ability θ which is common for all items, or a subinterval specified for the particular item

characteristic function. Let c_{g1} and c_{g2} denote the lower and upper asymptotes of the item characteristic function. Thus we can write

$$(2.2) \qquad \begin{cases} \lim_{\theta \to \underline{\theta}} P_g(\theta) = c_{g1} \\ \lim_{\theta \to \bar{\theta}} P_g(\theta) = c_{g2} \ , \end{cases}$$

where

$$(2.3) \qquad 0 \leq c_{g1} < c_{g2} \leq 1 \ .$$

We shall call the set of models, in which $c_{g1} = 0$ and $c_{g2} = 1$, Type A . Such models as the normal ogive model (Lord and Novick, 1968), the logistic model (Birnbaum, 1968), the linear model (Lazarsfeld, 1959) and the constant information model (Samejima, RR-79-1), belong to Type A . Note that in the former two models the interval $(\underline{\theta}, \bar{\theta})$ is the total range of ability θ , while in the latter two models it is a subinterval specified for each individual test item. In contrast to Type A , the set of models in which c_{g1} is greater than zero and c_{g2} is less than unity is called Type D . In addition to the above two types of models, we consider Type B and Type C , in which c_{g1} is greater than zero and $c_{g2} = 1$, and $c_{g1} = 0$ and c_{g2} is less than unity, respectively. They are summarized in Table 2-1.

Figure 2-1 presents examples of the item characteristic functions of the above four types of models. In these four examples, the item characteristic functions are given by

$$(2.4) \qquad P_g(\theta) = c_{g1} + (c_{g2}-c_{g1}) \Psi_g(\theta) \ ,$$

TABLE 2-1

Relationship between the Lower and Upper Asymptotes of the Item Characteristic Function for Each of the Four Types of Models for Binary Test Items.

Type A :	$0 = c_{g1} < c_{g2} = 1$
Type B :	$0 < c_{g1} < c_{g2} = 1$
Type C :	$0 = c_{g1} < c_{g2} < 1$
Type D :	$0 < c_{g1} < c_{g2} < 1$

where $\Psi_g(\theta)$ is the item characteristic function of the normal ogive model, which is given by

$$(2.5) \qquad \Psi_g(\theta) = (2\pi)^{-1/2} \int_{-\infty}^{a_g(\theta - b_g)} e^{-u^2/2}\, du \ ,$$

with the parameters, $a_g = 1.0$ and $b_g = 0.0$. For Types B and D , we have $c_{g1} = 0.2$, and, for Types C and D , $c_{g2} = 0.8$. We notice that the example for Type A which is given in Figure 2-1 is the normal ogive model, and that for Type B is the three-parameter normal ogive model.

We have no specific models of Types C and D which are commonly used, although the author has come across some researchers who have thought of Type C models. In the present paper, however, we include both Type C and Type D , in order to investigate the effect of noise which affects the upper asymptote of the item characteristic function as well as the lower asymptote.

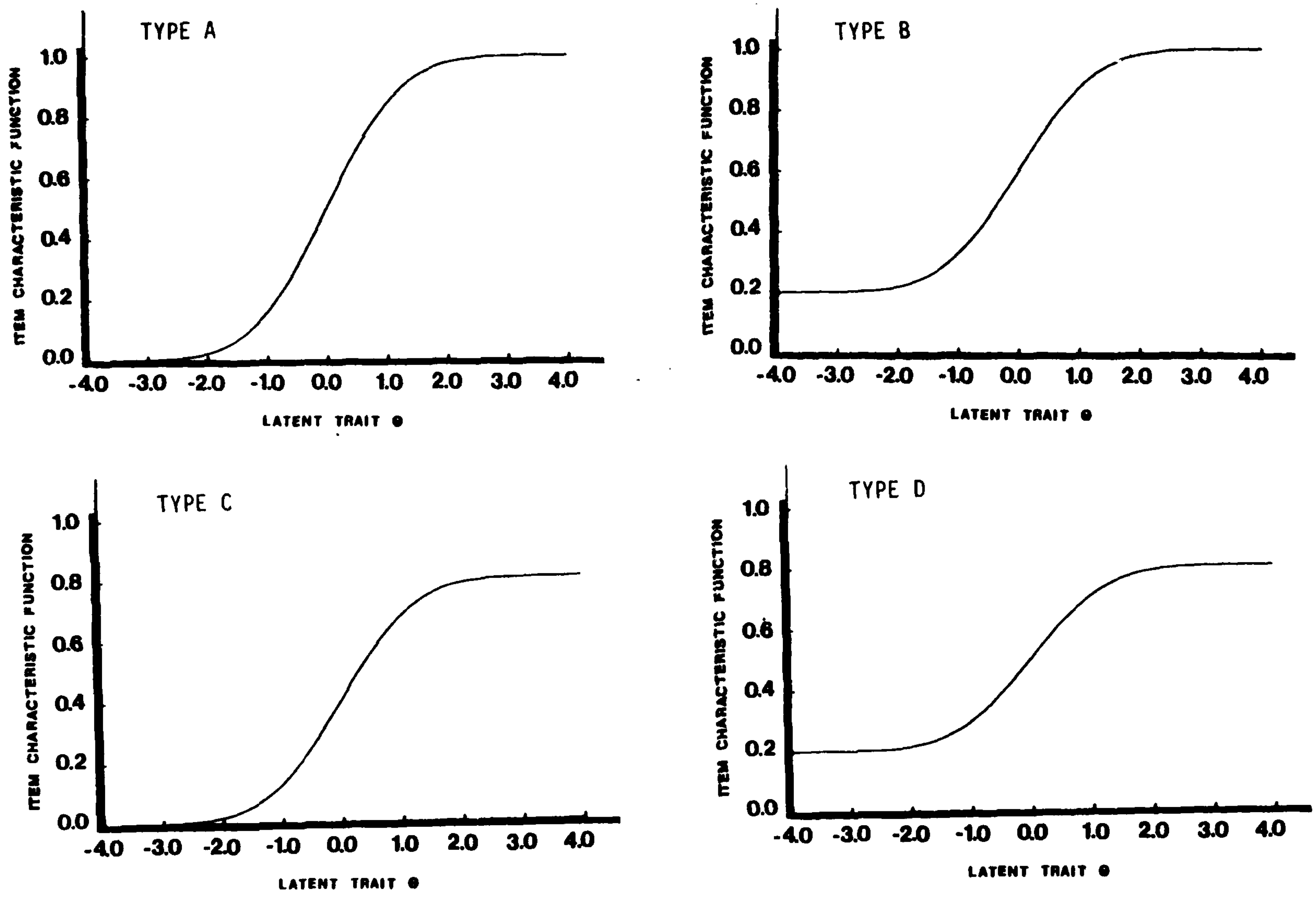


FIGURE 2-1

Examples of item characteristic functions of Types A, B, C and D .

III Information Loss

The item information function, as well as the test information function, has been used as a measure of accuracy in estimating the examinee's ability, which is specified locally, or for fixed values of ability θ . This characteristic of the item information function enables us to compare two or more different items with respect to their local accuracy in ability estimation, and to decide which item is best for a particular ability level. In spite of this fact, it has been pointed out (Samejima, RR-79-1) that there exists some constancy in item information among all items which belong to the same type.

Let τ be a strictly increasing transformation of θ such that

$$(3.1) \qquad \tau = \tau(\theta) \ .$$

We can write for the item information function, $I_g^*(\tau)$, of item g on the transformed latent trait

$$(3.2) \qquad I_g^*(\tau) = I_g(\theta)\,[\frac{d\theta}{d\tau}]^2 \ .$$

Thus we obtain

$$(3.3) \qquad \int_{\underline{\tau}}^{\bar{\tau}} [I_g^*(\tau)]^{1/2}\, d\tau = \int_{\underline{\theta}}^{\bar{\theta}} [I_g(\theta)]^{1/2}\, d\theta \ ,$$

where $\underline{\tau}$ and $\bar{\tau}$ are given by

$$(3.4) \qquad \begin{cases} \underline{\tau} = \tau(\underline{\theta}) \\ \bar{\tau} = \tau(\bar{\theta}) \ . \end{cases}$$

We notice that an item characteristic function of a specified model with given parameter values can be transformed to another, which belongs to the same model but with different values of parameters, by a strictly increasing transformation of the latent trait θ . Thus (3.3) implies that the area under the curve of the square root of the item information function is constant for all the item characteristic functions which belong to the same model, although each item may have a different amount of information for any fixed value of θ . This constancy of item information is expanded to all the item characteristic functions which belong to different models of the same type. It has been shown that this area equals π for all the models of Type A .

It should be noted that the relationship given in (3.3) can be expanded for any subinterval of $(\underline{\theta}, \bar{\theta})$. Let $(\underline{\underline{\theta}}, \bar{\bar{\theta}})$ be an arbitrary subinterval of $(\underline{\theta}, \bar{\theta})$. Thus we have

$$(3.5) \qquad (\underline{\underline{\theta}}, \bar{\bar{\theta}}) \leq (\underline{\theta}, \bar{\theta}) \ .$$

We can write

$$(3.6) \qquad \int_{\underline{\underline{\tau}}}^{\bar{\bar{\tau}}} [I_g^*(\tau)]^{1/2}\, d\tau = \int_{\underline{\underline{\theta}}}^{\bar{\bar{\theta}}} [I_g(\theta)]^{1/2}\, d\theta \ ,$$

where $\underline{\underline{\tau}}$ and $\bar{\bar{\tau}}$ are given by

$$(3.7) \qquad \begin{cases} \underline{\underline{\tau}} = \tau(\underline{\underline{\theta}}) \\ \bar{\bar{\tau}} = \tau(\bar{\bar{\theta}}) \ . \end{cases}$$

Equation (3.6) also implies that the constancy of item information applies

for different models which belong to different types. In this case, however, the areas are not necessarily the total areas but those for specified subintervals. To give an example, let us consider the four item characteristic functions of Types A , B , C and D , which are given in the preceding chapter. If we set $\underline{\underline{\tau}} = -\infty$ and $\overline{\overline{\tau}} = \infty$ for each of the last three item characteristic functions of Types B , C and D , then the corresponding values of $\underline{\underline{\theta}}$ and $\overline{\overline{\theta}}$ for the first item characteristic function, i.e., that of the normal ogive model with the parameters, $a_g = 1.0$ and $b_g = 0.0$, are: 1) $\underline{\underline{\theta}} = \Psi^{-1}(c_{g1})$ and $\overline{\overline{\theta}} = \infty$ for Type B , 2) $\underline{\underline{\theta}} = -\infty$ and $\overline{\overline{\theta}} = \Psi_g^{-1}(c_{g2})$ for Type C , and 3) $\underline{\underline{\theta}} = \Psi_g^{-1}(c_{g1})$ and $\overline{\overline{\theta}} = \Psi_g^{-1}(c_{g2})$ for Type D . Thus when $c_{g1} = 0.2$ and $c_{g2} = 0.8$, as is illustrated in Figure 2-1, we obtain $\Psi_g^{-1}(c_{g1}) \doteq -0.84$ and $\Psi_g^{-1}(c_{g2}) \doteq 0.84$.

It has been pointed out (Samejima, 1972, 1973b) that models of Type B , such as the three-parameter logistic model and the three-parameter normal ogive model, provide us with subintervals of θ for which the item response information function for $u_g = 1$ assumes negative values. It can be observed easily that the same is true for those which belong to Type C or Type D . It is questionable, therefore, that the item information function is as meaningful for those models of Types B and C as it is for those which belong to Type A and whether it assures a unique maximum for the likelihood function for each and every response pattern, as is the case with the normal ogive and logistic models (Samejima, 1969, 1972). For the moment, however, let us keep this question open, and proceed to investigate the item information loss caused by different characteristics of models. Let h be a binary test item which belongs

to Type A . We shall transform the latent trait θ to τ in such a way that

(3.8) $$\tau = \tau(\theta) = P_h^{*^{-1}}[P_g(\theta)] ,$$

where $P_h^*(\tau)$ is the item characteristic function of Type A defined on τ . Thus τ is a strictly increasing function of θ with the range

(3.9) $$\underline{\underline{\tau}} < \tau < \overline{\overline{\tau}} ,$$

where

(3.10) $$\begin{cases} \underline{\underline{\tau}} = P_h^{*^{-1}}[P_g(\underline{\theta})] = P_h^{*^{-1}}(c_{g1}) \\ \overline{\overline{\tau}} = P_h^{*^{-1}}[P_g(\overline{\theta})] = P_h^{*^{-1}}(c_{g2}) . \end{cases}$$

For convenience, and without loss of generality, let h follow the logistic model, such that

(3.11) $$P_h^*(\tau) = [1 + \exp\{-Da_h(\tau-b_h)\}]^{-1} .$$

Thus we have

(3.12) $$\begin{cases} \underline{\underline{\tau}} = [Da_h]^{-1}[\log c_{g1} - \log(1-c_{g1})] + b_h \\ \overline{\overline{\tau}} = [Da_h]^{-1}[\log c_{g2} - \log(1-c_{g2})] + b_h . \end{cases}$$

We can see from (3.12) that, if $c_{g1} = 0$, then $\underline{\underline{\tau}} = -\infty$ or is finite and, if $c_{g2} = 1$, then $\overline{\overline{\tau}} = \infty$ or is finite. We obtain for the total information Q

$$(3.13)\qquad Q = \int_{\underline{\theta}}^{\bar{\theta}} [I_g(\theta)]^{1/2}\, d\theta = Da_h \int_{\underline{\underline{\tau}}}^{\bar{\bar{\tau}}} [\exp\{Da_h(\tau-b_h)\}]^{1/2} \cdot [1 + \exp\{Da_h(\tau-b_h)\}]^{-1}\, d\tau \ .$$

Now we shall further define τ^* such that

$$(3.14)\qquad \tau^* = \tau^*(\tau) = [\exp\{Da_h(\tau-b_h)\}]^{1/2} \ .$$

Thus we have

$$(3.15)\qquad \frac{d\tau}{d\tau^*} = 2(Da_h\tau^*)^{-1} \ ,$$

and from (3.12) and (3.14) we can write

$$(3.16)\qquad \underline{\underline{\tau}}^* = \tau^*(\underline{\underline{\tau}}) = c_{g1}{}^{1/2}(1-c_{g1})^{-1/2}$$

and

$$(3.17)\qquad \bar{\bar{\tau}}^* = \tau^*(\bar{\bar{\tau}}) = c_{g2}{}^{1/2}(1-c_{g2})^{-1/2} \ .$$

We obtain from (3.13), (3.14), (3.15), (3.16) and (3.17)

$$(3.18)\qquad Q = 2[\tan^{-1}\{c_{g2}/(1-c_{g2})\}^{1/2} - \tan^{-1}\{c_{g1}/(1-c_{g1})\}^{1/2}] \ .$$

It is obvious from (3.18) that, when item g belongs to Type A , i.e., $c_{g1} = 0$ and $c_{g2} = 1$, as is the case with the normal ogive model or with the logistic model on the dichotomous response level, this quantity, Q , assumes the maximal value, with

$$Q = \pi \ , \tag{3.19}$$

and the result is consistent with our previous finding. We can also see from (3.18) that, as c_{g1} departs from zero, and c_{g2} from unity, the total information Q becomes progressively smaller than π. In the three examples shown in Figure 2-1, we obtain $Q \doteq 2.214$ for both Types B and D, and $Q \doteq 1.287$ for Type D. We can easily see from (3.18) that the total information Q assumes the same value for Types B and C whenever $c_{g1} = 1 - c_{g2}$.

IV Three-Parameter Logistic Model

The three-parameter logistic model may be the model of Type B which is most frequently used by researchers. The logistic model was originally developed as a substitute for the normal ogive model because of its closeness in shape and the mathematical simplicity which provides us with a simple sufficient statistic for the response pattern V (cf. Birnbaum, 1968). Following the knowledge or random guessing principle, the third paramter, c_g, which is called the guessing parameter, was added to the model to change it to the three-parameter logistic model. Thus we have for the item characteristic function of the three-parameter logistic model

$$P_g(\theta) = c_g + (1-c_g)\ \Psi_g(\theta) \tag{4.1}$$

where

$$\Psi_g(\theta) = [1 + e^{-Da_g(\theta-b_g)}]^{-1} \ . \tag{4.2}$$

The function given by (4.2) is the item characteristic function in the logistic model, which differs by no greater than 0.1 from the item characteristic function of the normal ogive model given by (2.5), throughout the whole range of θ, if we set the scaling factor D equal to 1.7 . It is obvious from (4.1) that the three-parameter logistic model belongs to Type B , with $c_{g1} = c_g$.

From (3.18) we can write for the total item information for the three-parameter logistic model

$$(4.3) \qquad Q = \pi - 2\tan^{-1}[c_g/(1-c_g)]^{1/2} .$$

Since the total item information Q for any model of Type A is π , the second term on the right hand side of (4.3) indicates the information loss caused by the lower asymptote c_g , in comparison with Q in any model of Type A . When the multiple-choice test item has five alternatives, $c_g = 1/5 = 0.20$, and the information loss is as large as 0.295π . When it has four alternatives, $c_g = 1/4 = 0.25$, and the information loss is 0.333π , which incicates that one-third of the total information is lost because of the noise caused by random guessing.

Figure 4-1 illustrates the information loss caused by the noise in the three-parameter logistic model. In this figure, the square root of the item information function of item h , which follows the logistic model on the latent trait τ with the parameters, $a_h = 1.0$ and $b_h = 0.0$, and the scaling factor, $D = 1.7$, is drawn. Thus we can write

$$(4.4) \qquad P_h^*(\tau) = [1 + e^{-Da_h(\tau-b_h)}]^{-1} = [1 + e^{-1.7\tau}]^{-1} ,$$

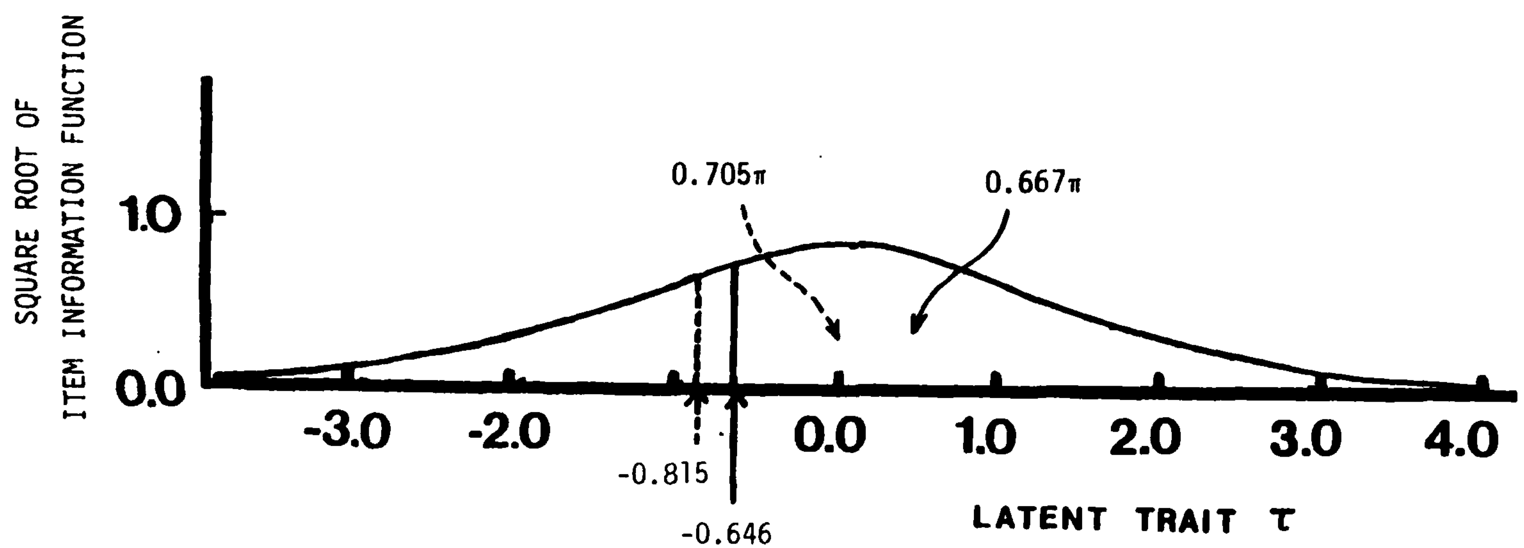


FIGURE 4-1

Information Loss Caused by the Lower Asymptote c_g of the Item Characteristic Function in the Three-Parameter Logistic Model for $c_g = 0.20$ (Dashes) and for $c_g = 0.25$ (Solid Line).

and

$$(4.5)\qquad I_h^*(\tau)^{1/2} = Da_g\, P_h^*(\tau)^{1/2}\, [1-P_h^*(\tau)]^{1/2} = 1.7(2+e^{-1.7\tau}+e^{1.7\tau})^{-1/2} \ .$$

The total area under the solid curve in Figure 4-1 equals π , and the area which lies on the left hand side of the vertical, dashed line indicates the information loss caused by the lower asymptote, $c_g = 0.20$, and that of the vertical, solid line corresponds to the information loss caused by $c_g = 0.25$. Table 4-1 presents these values and the corresponding information loss for each of the four other cases in which $c_g = 0.100$, 0.125 , 0.333 and 0.500 , respectively. In the same table also presented are the value of $\underline{\underline{\tau}}$, the critical value θ_g below which the item response information function for $u_g = 1$ assumes negative values, and the values of $\Psi_g(\theta)$, $P_g(\theta)$ and $I_g(\theta)$ at $\theta = \theta_g$ for each case, with $D = 1.7$, $a_g = a_h = 1.0$ and $b_g = b_h = 0.0$. We can write for the last four columns

$$(4.6)\qquad \theta_g = (2Da_g)^{-1} \log c_g + b_g \ ,$$

$$(4.7)\qquad \Psi_g(\theta_g) = [1 + \exp\{-Da_g(\theta_g - b_g)\}]^{-1} = c_g^{1/2}(1+c_g^{1/2})^{-1} \ ,$$

$$(4.8)\qquad P_g(\theta_g) = c_g + (1-c_g)\Psi_g(\theta_g) = c_g^{1/2} \ ,$$

and

$$I_g(\theta_g) = (1-c_g)D^2 a_g^{\,2}\{\Psi_g(\theta)\}^2[1-\Psi_g(\theta_g)][c_g+(1-c_g)\Psi_g(\theta_g)]^{-1} \tag{4.9}$$

$$= D^2 a_g^{\,2}\, c_g^{\,1/2}(1-c_g^{\,1/2})(1+c_g^{\,1/2})^{-2} \; .$$

TABLE 4-1

Information Loss Caused by the Lower Asymptote of the Item Characteristic Function in the Three-Parameter Logistic Model for Each of the Six Values of the Asymptote c_g , Together with $\underline{\underline{\tau}}$, the Critical Value θ_g and the Logistic Function $\Psi_g(\theta)$, the Item Characteristic Function $P_g(\theta)$ and the Item Information Function $I_g(\theta)$ at $\theta = \theta_g$.

c_g	$\underline{\underline{\tau}}$	Information Loss	θ_g	$\Psi_g(\theta_g)$	$P_g(\theta_g)$	$I_g(\theta_g)$
0.100	-1.292	0.644 (20.5%)	-0.677	0.240	0.361	0.361
0.125	-1.145	0.723 (23.0%)	-0.612	0.261	0.354	0.361
0.200	-0.815	0.927 (29.5%)	-0.473	0.309	0.447	0.341
0.250	-0.646	1.047 (33.3%)	-0.408	0.333	0.500	0.321
0.333	-0.408	1.231 (39.2%)	-0.323	0.366	0.577	0.283
0.500	0.000	1.571 (50.0%)	-0.204	0.414	0.707	0.205

V Item Response Information Functions

There are certain models for binary items which assure the existence of a unique maximum for the likelihood function of every possible response pattern, such as the normal ogive model, the logistic model, etc. In fact, except for the two extreme response patterns in which $u_g = 0$ for all the items and $u_g = 1$ for all the items, respectively, the likelihood function has a unique local maximum in those models. It has been pointed out (Samejima, 1969, 1972) that a sufficient condition for the unique maximum is: 1) that the basic function, which is defined by (1.1), is strictly decreasing in θ throughout its whole range, and 2) that its upper asymptote is non-negative and its lower asymptote is non-positive. For brevity, we shall call it the unique maximum condition. This condition implies that the item response information function is positive except, at most, at an enumerable number of points of θ .

It has been shown (Samejima, 1972, 1973b) that the three-parameter logistic model does not satisfy the unique maximum condition, and that the likelihood function for certain response patterns has more than one modal point. The same is true with the three-parameter normal ogive model. This is one of the serious problems which are caused by noise accomodated by a three-parameter model.

In this chapter, we solely consider item characteristic functions given by (2.4) in which $\Psi_g(\theta)$ is the item characteristic function of Type A that satisfies the unique maximum condition. For simplicity, let $\Psi'_g(\theta)$ denote the first partial derivative of $\Psi_g(\theta)$ with respect to θ , and $\Psi''_g(\theta)$ be the second partial derivative, so that we have

$$(5.1)\qquad \Psi'_g(\theta) = \frac{\partial}{\partial\theta}\Psi_g(\theta)$$

and

$$(5.2)\qquad \Psi''_g(\theta) = \frac{\partial^2}{\partial\theta^2}\Psi_g(\theta)\ .$$

We can write from (1.1) and (2.4) for the basic function

$$(5.3)\qquad A_{u_g}(\theta)\begin{cases} = -(c_{g2}-c_{g1})\Psi'_g(\theta)[(1-c_{g1})-(c_{g2}-c_{g1})\Psi_g(\theta)]^{-1} & u_g = 0 \\ = (c_{g2}-c_{g1})\Psi'_g(\theta)[c_{g1}+(c_{g2}-c_{g1})\Psi_g(\theta)]^{-1} & u_g = 1\ . \end{cases}$$

Substituting (5.3) into (1.6), we obtain for the item response information function

$$(5.4)\qquad I_{u_g}(\theta)\begin{cases} = [(c_{g2}-c_{g1})^2[\{\Psi'_g(\theta)\}^2+\{1-\Psi_g(\theta)\}\Psi''_g(\theta)]+(1-c_{g2}) \\ \quad (c_{g2}-c_{g1})\Psi''_g(\theta)][(1-c_{g1})-(c_{g2}-c_{g1})\Psi_g(\theta)]^{-2} & u_g = 0 \\ = [(c_{g2}-c_{g1})^2[\{\Psi'_g(\theta)\}^2-\Psi_g(\theta)\Psi''_g(\theta)]-c_{g1}(c_{g2}-c_{g1})\Psi''_g(\theta)] \\ \quad [c_{g1}+(c_{g2}-c_{g1})\Psi_g(\theta)]^{-2} & u_g = 1\ . \end{cases}$$

For the models of Type A in which $c_{g1} = 0$ and $c_{g2} = 1$, both (5.3) and (5.4) are simplified in the forms

$$(5.5)\qquad A_{u_g}(\theta)\begin{cases} = -\Psi_g'(\theta)[1-\Psi_g(\theta)]^{-1} & u_g = 0 \\ = \Psi_g'(\theta)[\Psi_g(\theta)]^{-1} & u_g = 1\ , \end{cases}$$

and

$$(5.6)\qquad I_{u_g}(\theta)\begin{cases} = [\{\Psi_g'(\theta)\}^2+\{1-\Psi_g(\theta)\}\Psi_g''(\theta)][1-\Psi_g(\theta)]^{-2} & u_g = 0 \\ = [\{\Psi_g'(\theta)\}^2-\Psi_g(\theta)\Psi_g''(\theta)]\{\Psi_g(\theta)\}^{-2} & u_g = 1\ . \end{cases}$$

It is obvious from (5.5) that the sign is reversed for the two $A_{u_g}(\theta)$'s for any fixed value of θ . Since we are solely dealing with the item characteristic functions which are strictly increasing in θ , and $\Psi_g(\theta)$ is such an item characteristic function of Type A , we can say that $A_{u_g}(\theta)$ assumes negative values for $u_g = 0$ except, at most, at an enumerable number of points of θ where it is zero, and it assumes positive values for $u_g = 1$ except, at most, at an enumerable number of points of θ . Since we also deal solely with $\Psi_g(\theta)$ which satisfies the unique maximum condition, $A_{u_g}(\theta)$ is strictly decreasing in θ and $I_{u_g}(\theta)$ is non-negative throughout the whole range of θ . In fact, in models such as the normal ogive model and the logistic model, $I_{u_g}(\theta)$ is positive everywhere both for $u_g = 0$ and $u_g = 1$. The upper and lower asymptotes of $A_{u_g}(\theta)$ for $u_g = 0$ in the normal ogive model are zero and negative infinity, and those for $u_g = 1$ are positive infinity and zero, respectively. On the other hand, the two asymptotes for $u_g = 0$ in the logistic model are zero and $-Da_g$, and those for $u_g = 1$ are Da_g and zero, respectively, the outcome which is somewhat different from the one in the normal ogive model, in spite of its similarity

in the shape of the item characteristic function (cf. Samejima, 1969, 1972). It is obvious that both models satisfy the unique maximum condition.

There are models of Type A which do not satisfy the unique maximum condition. Consider, for example, the linear model. Its item characteristic function, $P_g(\theta)$, is given by

$$(5.7)\qquad P_g(\theta)\begin{cases} = (\theta-\alpha_g)(\beta_g-\alpha_g)^{-1} & \alpha_g \leq \theta \leq \beta_g \\ = 0 & \text{otherwise} . \end{cases}$$

It is obvious that this model belongs to Type A. We have for the first and second derivatives of $P_g(\theta)$ with respect to θ for the interval, $\alpha_g < \theta < \beta_g$,

$$(5.8)\qquad \frac{\partial}{\partial\theta} P_g(\theta) = (\beta_g-\alpha_g)^{-1}$$

and

$$(5.9)\qquad \frac{\partial^2}{\partial\theta^2} P_g(\theta) = 0$$

respectively. Replacing $\Psi_g(\theta)$ and its derivatives in (5.5) and (5.6) by the above $P_g(\theta)$ and its corresponding derivatives, we obtain

$$(5.10)\qquad A_{u_g}(\theta)\begin{cases} = -(\beta_g-\theta)^{-1} & u_g = 0 \\ = (\theta-\alpha_g)^{-1} & u_g = 1 \end{cases}$$

and

$$(5.11) \qquad I_{u_g}(\theta) \begin{cases} = (\beta_g-\theta)^{-2} > 0 & u_g = 0 \\ = (\theta-\alpha_g)^2 > 0 & u_g = 1 . \end{cases}$$

We can see from (5.10) and (5.11) that, while the basic function is strictly decreasing in θ both for $u_g = 0$, and for $u_g = 1$, the upper and lower asymptotes at $\theta = \alpha_g$ and $\theta = \beta_g$ are $-(\beta_g-\alpha_g)$ and negative infinity, respectively, for $u_g = 0$, and those for $u_g = 1$ are positive infinity and $(\beta_g-\alpha_g)$. Thus it does not satisfy the unique maximum condition which requires that the upper asymptote for $u_g = 0$ be non-negative and the lower asymptote for $u_g = 1$ be non-positive. In addition to this fact, since the linear model provides us with a strictly increasing item characteristic function only for the interval, $\alpha_g \leq \theta \leq \beta_g$, and this interval varies from one item to another, we must, in general, restrict the interval of θ to the intersection of those individual intervals, and, consequently, the two asymptotes will possibly be further to the negative and positive sides, respectively.

This last statement is applicable even for models which provide us with item characteristic functions that satisfy the unique maximum condition, if the intervals for which the items are strictly increasing are defined individually and differ from one another, as is the case with the linear model. Take, for example, the constant information model (Samejima, RR-79-1, RR-79-3). We can write for the item characteristic function in the constant information model

$$(5.12) \qquad P_g(\theta) \begin{cases} = \sin^2[\gamma_g(\theta-\delta_g)+(\pi/4)] & -[\pi/(4\gamma_g)]+\delta_g \leq \theta \leq [\pi/(4\gamma_g)]+\delta_g \\ = 0 & \text{otherwise,} \end{cases}$$

where γ_g and δ_g are item discrimination and difficulty parameters, respectively. Substituting this for $\Psi_g(\theta)$ in (5.5) and (5.6) and rearranging, we can write for the basic function and the item response information function defined for the interval of θ, $-[\pi/(4\gamma_g)] < \theta < \pi/(4\gamma_g)$,

$$(5.13) \qquad A_{u_g}(\theta) \begin{cases} = -2\gamma_g \tan[\gamma_g(\theta-\delta_g)+(\pi/4)] & u_g = 0 \\ = 2\gamma_g \cot[\gamma_g(\theta-\delta_g)+(\pi/4)] & u_g = 1 , \end{cases}$$

and

$$(5.14) \qquad I_{u_g}(\theta) \begin{cases} = 2\gamma_g^{\;2} \sec^2[\gamma_g(\theta-\delta_g)+(\pi/4)] > 0 & u_g = 0 \\ = 2\gamma_g^{\;2} \csc^2[\gamma_g(\theta-\delta_g)+(\pi/4)] > 0 & u_g = 1 . \end{cases}$$

We can see from these two formulae that the basic function, $A_{u_g}(\theta)$, is strictly decreasing in θ both for $u_g = 0$ and for $u_g = 1$, and its upper and lower asymptotes are zero and negative infinity for $u_g = 0$ and positive infinity and zero for $u_g = 1$, respectively. Thus the unique maximum condition is satisfied for the above interval of θ. And yet it is not satisfied for every interval of θ, since the interval for which the item characteristic function is strictly increasing is defined individually and possibly differs from the one for another item. We must take the intersection of the n individual intervals, therefore, and for that subinterval of θ the unique maximum condition is not satisfied for every item, unless all the n items

are equivalent items. Note that, in such a case, the unique maximum likelihood estimate exists for every response pattern regardless of the model, and is obtained from the test score which is a simple sufficient statistic for the response pattern V .

For models of Type B , in which $c_{g2} = 1$, we can simplify (5.3) and (5.4) in the forms

$$
(5.15)\qquad A_{u_g}(\theta)\begin{cases} = -\Psi_g'(\theta)[1-\Psi_g(\theta)]^{-1} & u_g = 0 \\ = (1-c_{g1})\Psi_g'(\theta)[c_{g1}+(1-c_{g1})\Psi_g(\theta)]^{-1} & u_g = 1 \ , \end{cases}
$$

and

$$
(5.16)\qquad I_{u_g}(\theta)\begin{cases} = [\{\Psi_g'(\theta)\}^2+\{1-\Psi_g(\theta)\}\Psi_g''(\theta)][1-\Psi_g(\theta)]^{-2} & u_g = 0 \\ = [(1-c_{g1})^2[\{\Psi_g'(\theta)\}^2-\Psi_g(\theta)\Psi_g''(\theta)]-c_{g1}(1-c_{g1})\Psi_g''(\theta)] & \\ \qquad [c_{g1}+(1-c_{g1})\Psi_g(\theta)]^{-2} & u_g = 1 \ . \end{cases}
$$

Comparison of these formulae with (5.5) and (5.6) reveals that, for $u_g = 0$, the basic function, and hence the item response information function, are the same as those in models of Type A , while those for $u_g = 1$ have more complicated forms than the counterparts in models of Type A because of the additional terms and factors which include c_{g1} . It is obvious, therefore, that the basic function for $u_g = 0$ satisfies the unique maximum condition and the corresponding item response information function is positive except, at most, at an enumerable number of points of θ , as is the case with Type A . We can see from the second line of (5.15)

that the basic function for $u_g = 1$ is positive throughout the entire range of θ except, at most, at an enumerable number of points where it assumes zero, as is also the case with Type A . We can also see that, if the asymptote of $\Psi_g'(\theta)$ at the lower end of θ is zero, as is the case with such models as the normal ogive and logistic models, the asymptote of the basic function for $u_g = 1$ is also zero, and the function cannot be strictly decreasing in θ , and in such a case there must be a subset of θ for which the item response information function assumes negative values. A close examination of the second line of (5.16) reveals that the second term, $-c_{g1}(1-c_{g1})\Psi_g''(\theta)$, of the first factor on the right hand side of the formula takes an important role in specifying the subset of θ in which the item response information function for $u_g = 1$ is negative.

For models of Type C , in which $c_{g1} = 0$, (5.3) and (5.4) are simplified to become

$$(5.17) \qquad A_{u_g}(\theta) \begin{cases} = -c_{g2}\Psi_g'(\theta)[1-c_{g2}\Psi_g(\theta)]^{-1} & u_g = 0 \\ = \Psi_g'(\theta)[\Psi_g(\theta)]^{-1} & u_g = 1 , \end{cases}$$

and

$$(5.18) \qquad I_{u_g}(\theta) \begin{cases} = [c_{g2}{}^2(\{\Psi_g'(\theta)\}^2-\Psi_g(\theta)\Psi_g''(\theta))+c_{g2}\Psi_g''(\theta)][1-c_{g2}\Psi_g(\theta)]^{-2} & u_g = 0 \\ = [\{\Psi_g'(\theta)\}^2-\Psi_g(\theta)\Psi_g''(\theta)][\Psi_g(\theta)]^{-2} & u_g = 1 . \end{cases}$$

We can see from these formulae that, contrary to the case of models of Type B, for $u_g = 1$ both the basic function and the item response information functions are identical with those in the models of Type A, while for $u_g = 0$ they are different and more complicated from the counterparts in the models of Type A. Thus it is obvious that the basic function for $u_g = 1$ satisfies the unique maximum condition, and the corresponding item response information function is positive except, at most, for an enumerable number of points of θ where it assumes zero. Since $\Psi_g(\theta)$ is strictly increasing in θ, the basic function for $u_g = 0$ is non-positive, and, if the asymptote of the first derivative of $\Psi_g(\theta)$ at the upper end of θ is zero, as we find in the normal ogive model, in the logistic model, etc., the asymptote of the basic function at the upper end of θ is zero, so the basic function cannot be a strictly decreasing function of θ. The unique maximum condition is not satisfied, therefore, for $u_g = 0$. There exists a subset of θ for which the item response information function assumes negative values. A close examination of the first line of (5.18) reveals that the second term, $c_{g2}\Psi''_g(\theta)$, of the first factor on the right hand side of the formula takes an important role in determining the subset of θ in which the item response information function for $u_g = 0$ assumes negative values.

For models of Type D, (5.3) and (5.4) are directly applied. We can see from (5.3) that, if the first derivative of $\Psi_g(\theta)$ tends to zero at both the upper and lower ends of θ, as is the case with the normal ogive model, the logistic model, etc., the unique maximum condition is not satisfied either for $u_g = 0$ or for $u_g = 1$. There exists a subset

of θ in which the item response information function assumes negative values for each item score. For $u_g = 0$, the second term of the first factor, $(1-c_{g2})(c_{g2}-c_{g1})\Psi''_g(\theta)$, plays an important role in determining this subset of θ, and, for $u_g = 1$, so does the second term of the first factor, $-c_{g1}(c_{g2}-c_{g1})\Psi''_g(\theta)$.

When $\Psi_g(\theta)$ is given as the item characteristic function in the normal ogive model, we can write

$$(5.19) \qquad \Psi_g(\theta) = (2\pi)^{-1/2} \int_{-\infty}^{a_g(\theta-b_g)} e^{-u^2/2} du ,$$

$$(5.20) \qquad \Psi'_g(\theta) = a_g(2\pi)^{-1/2} \exp[-a_g^{\,2}(\theta-b_g)^2/2] ,$$

and

$$(5.21) \qquad \Psi''_g(\theta) = -a_g^{\,2}(\theta-b_g)\Psi'_g(\theta) = -a_g^{\,3}(2\pi)^{-1/2}(\theta-b_g)\exp[-a_g^{\,2}(\theta-b_g)^2/2] .$$

When we substitute them into (5.15) and (5.16), respectively, we have the basic function and the item response information function of the three-parameter normal ogive model.

If we specify that $\Psi_g(\theta)$ be the item characteristic function in the logistic model, then we will have

$$(5.22) \qquad \Psi_g(\theta) = [1+e^{-Da_g(\theta-b_g)}]^{-1} ,$$

$$(5.23) \qquad \Psi'_g(\theta) = Da_g\Psi_g(\theta)[1-\Psi_g(\theta)] ,$$

and

$$\Psi''_g(\theta) = Da_g \Psi'_g(\theta)[1-2\Psi_g(\theta)] = D^2 a_g{}^2 \Psi_g(\theta)[1-\Psi_g(\theta)][1-2\Psi_g(\theta)] \ . \tag{5.24}$$

Substitution of (5.22) into (2.4) provides us with curves for the item characteristic functions of Types A , B , C and D which are similar to those which are illustrated in Figure 2-1, in which the item characteristic function of the normal ogive model is adopted for $\Psi_g(\theta)$. If we substitute (5.22) through (5.24) into (5.5) and (5.6), then we will obtain the basic function and the item response information function in the logistic model. Thus we can write

$$A_{u_g}(\theta) \begin{cases} = -Da_g \Psi_g(\theta) & u_g = 0 \\ = Da_g[1-\Psi_g(\theta)] & u_g = 1 \end{cases} \tag{5.25}$$

and

$$I_{u_g}(\theta) \begin{cases} = D^2 a_g{}^2 \Psi_g(\theta)[1-\Psi_g(\theta)] > 0 & u_g = 0 \\ = D^2 a_g{}^2 \Psi_g(\theta)[1-\Psi_g(\theta)] > 0 & u_g = 1 \end{cases} \tag{5.26}$$

It is obvious from (5.25) and (5.26) that the basic function is strictly decreasing in θ and the item response information function is positive throughout the entire range of θ , both for $u_g = 0$ and for $u_g = 1$. Since the item response information function is the same for $u_g = 0$ and $u_g = 1$, its conditional expectation, i.e., the item information function $I_g(\theta)$, is also identical with the item response information function. Figure 5-1 illustrates this curve, which represents the item response information function for $u_g = 0$ and $u_g = 1$ and the item information function, with the item parameters, $a_g = 1.00$ and $b_g = 0.00$, and the scaling factor, $D = 1.7$. Since this example is of Type A , we have $c_{g1} = 0.000$ and $c_{g2} = 1.000$ as the third and fourth parameters.

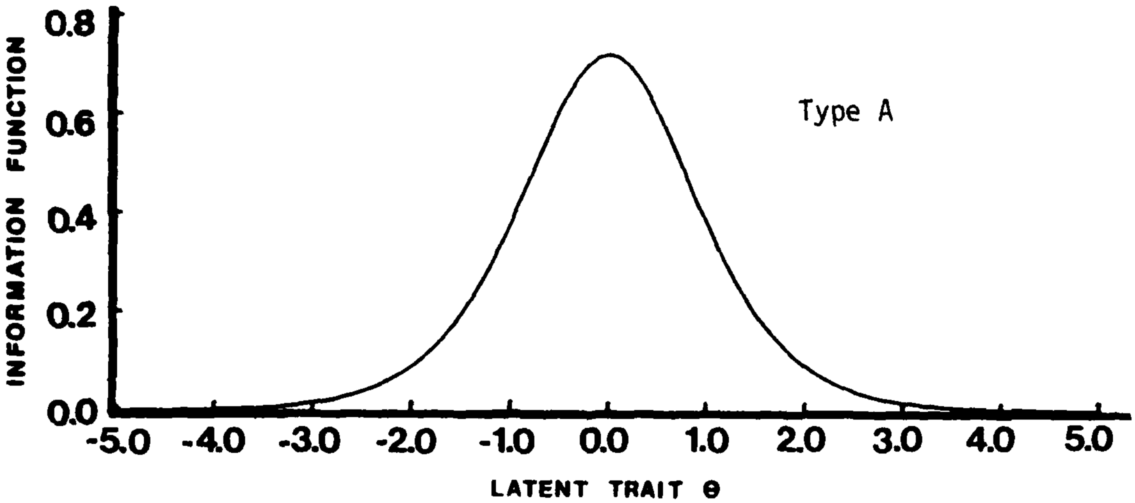


FIGURE 5-1

Three Identical Curves for the Item Response Information Function for $u_g = 0$ and $u_g = 1$ and the Item Information Function in the Logistic Model with $D = 1.7$, $a_g = 1.00$ and $b_g = 0.00$, as an Example of the Type A Model .

As was observed earlier, for models of the other three types, Types B , C and D , one of the two curves, or both, of the item response information function has a subset of θ for which it assumes negative values. Substitution of (5.22) through (5.24) into (5.3) and (5.4) provides us with the basic function and the item response information function for models of Type D in the forms

$$(5.27) \qquad A_{u_g}(\theta) \begin{cases} = -(c_{g2}-c_{g1})Da_g\Psi_g(\theta)[1-\Psi_g(\theta)][(1-c_{g1})-(c_{g2}-c_{g1})\Psi_g(\theta)]^{-1} & u_g = 0 \\ = (c_{g2}-c_{g1})Da_g\Psi_g(\theta)[1-\Psi_g(\theta)][c_{g1}+(c_{g2}-c_{g1})\Psi_g(\theta)]^{-1} & u_g = 1 \ , \end{cases}$$

and

$$(5.28) \qquad I_{u_g}(\theta) \begin{cases} = (c_{g2}-c_{g1})D^2a_g^2\Psi_g(\theta)[1-\Psi_g(\theta)][(1-c_{g1})\{1-\Psi_g(\theta)\}^2-(1-c_{g2})\{\Psi_g(\theta)\}^2] \\ \qquad [(1-c_{g1})-(c_{g2}-c_{g1})\Psi_g(\theta)]^{-2} & u_g = 0 \\ = (c_{g2}-c_{g1})D^2a_g^2\Psi_g(\theta)[1-\Psi_g(\theta)][c_{g2}\{\Psi_g(\theta)\}^2-c_{g1}\{1-\Psi_g(\theta)\}^2] \\ \qquad [c_{g1}+(c_{g2}-c_{g1})\Psi_g(\theta)]^{-2} & u_g = 1 \ , \end{cases}$$

respectively. For Type B , in which $c_{g2} = 1.000$, (5.27) and (5.28) are further simplified, and both the basic function and the item response information for $u_g = 0$ are the same as those for Type A , which are given as the first lines of (5.25) and (5.26), respectively, whereas for $u_g = 1$ we obtain

$$(5.29) \qquad A_{u_g}(\theta) = (1-c_{g1})Da_g\Psi_g(\theta)[1-\Psi_g(\theta)][c_{g1}+(1-c_{g1})\Psi_g(\theta)]^{-1} \qquad u_g = 1$$

and

$$(5.30) \qquad I_{u_g}(\theta) = (1-c_{g1})D^2a_g{}^2\Psi_g(\theta)[1-\Psi_g(\theta)][\{\Psi_g(\theta)\}^2-c_{g1}\{1-\Psi_g(\theta)\}^2] [c_{g1}+(1-c_{g1})\Psi_g(\theta)]^{-2} \qquad u_g = 1 .$$

For Type C , in which $c_{g1} = 0.000$, both the basic function and the item response information function for $u_g = 1$ are identical with those for Type A , which are given as the second lines of (5.25) and (5.26), respectively, and for $u_g = 0$ we can write

$$(5.31) \qquad A_{u_g}(\theta) = -c_{g2}Da_g\Psi_g(\theta)[1-\Psi_g(\theta)][1-c_{g2}\Psi_g(\theta)]^{-1} \qquad u_g = 0$$

and

$$(5.32) \qquad I_{u_g}(\theta) = c_{g2}D^2a_g{}^2\Psi_g(\theta)[1-\Psi_g(\theta)][\{1-\Psi_g(\theta)\}^2-(1-c_{g2})\{\Psi_g(\theta)\}^2] [1-c_{g2}\Psi_g(\theta)]^{-2} \qquad u_g = 0 .$$

Figure 5-2 presents the three sets of the item response information and the item information functions of Type B . In this figure, the two curves for the item response information function are drawn by solid lines, and the one for the item information function is shown by a dashed line. Note that they are the information functions in the three-parameter logistic model. The item parameters adopted here are, again,

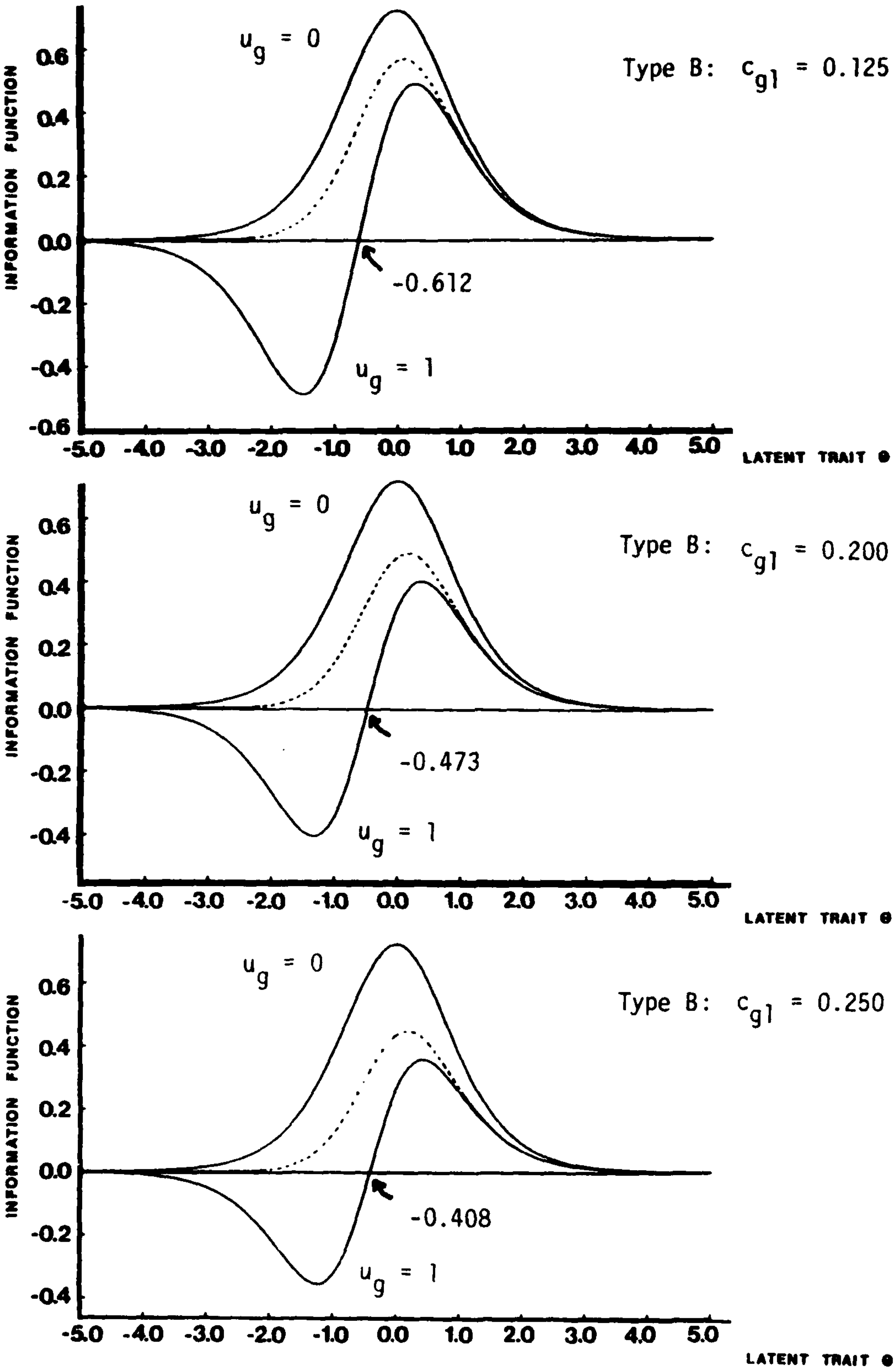


FIGURE 5-2

Item Response Information Function (Solid Lines) and Item Information Function (Dashed Line) of Type B Model: Logistic Function Is Used for $\Psi_g(\theta)$ with $a_g = 1.00$, $D = 1.7$ and $b_g = 0.00$, and specified c_{g1}.

$a_g = 1.00$ and $b_g = 0.000$, and the third parameter, c_{g1} , equals 0.125 in the first graph, 0.200 in the second graph and 0.250 in the third graph, respectively, and the scaling factor D is set equal to 1.7 . Note that these three values correspond to the guessing parameters when there are eight, five and four alternatives in the multiple-choice test item, respectively. As was observed in a previous study (Samejima, 1973b), there is a subinterval on the negative side of θ for which the item response information function assumes negative values for $u_g = 1$.

In contrast to the above result, the corresponding information functions for Type C are illustrated in Figure 5-3. Both the item discrimination parameter, a_g , and the item difficulty parameter, b_g , as well as the scaling factor D , are the same as in the preceding examples, with $c_{g2} = 0.875$ in the first graph, $c_{g2} = 0.800$ in the second graph, and $c_{g2} = 0.750$ in the third graph, respectively. We can see that, in each of these three examples of Type C , there is a subinterval on the positive side of θ for which the item response information function assumes negative values for $u_g = 0$.

Figure 5-4 presents nine examples of the corresponding sets of information functions for Type D . In these examples, both the item discrimination parameter a_g and the item difficulty parameter b_g , and the scaling factor D , are the same as those in the preceding examples, and all the nine possible pairs of $c_{g1} = 0.125, 0.200, 0.250$ and $c_{g2} = 0.875, 0.800, 0.750$ are adopted. In the first three graphs, there is the relationship, $c_{g1} = 1 - c_{g2}$, and the resultant two curves of the item response information function are symmetric with $\theta = 0.000$ as the axis of symmetry, while in the other six graphs, the above relationship does not hold between c_{g1} and c_{g2} , and the two curves of item response

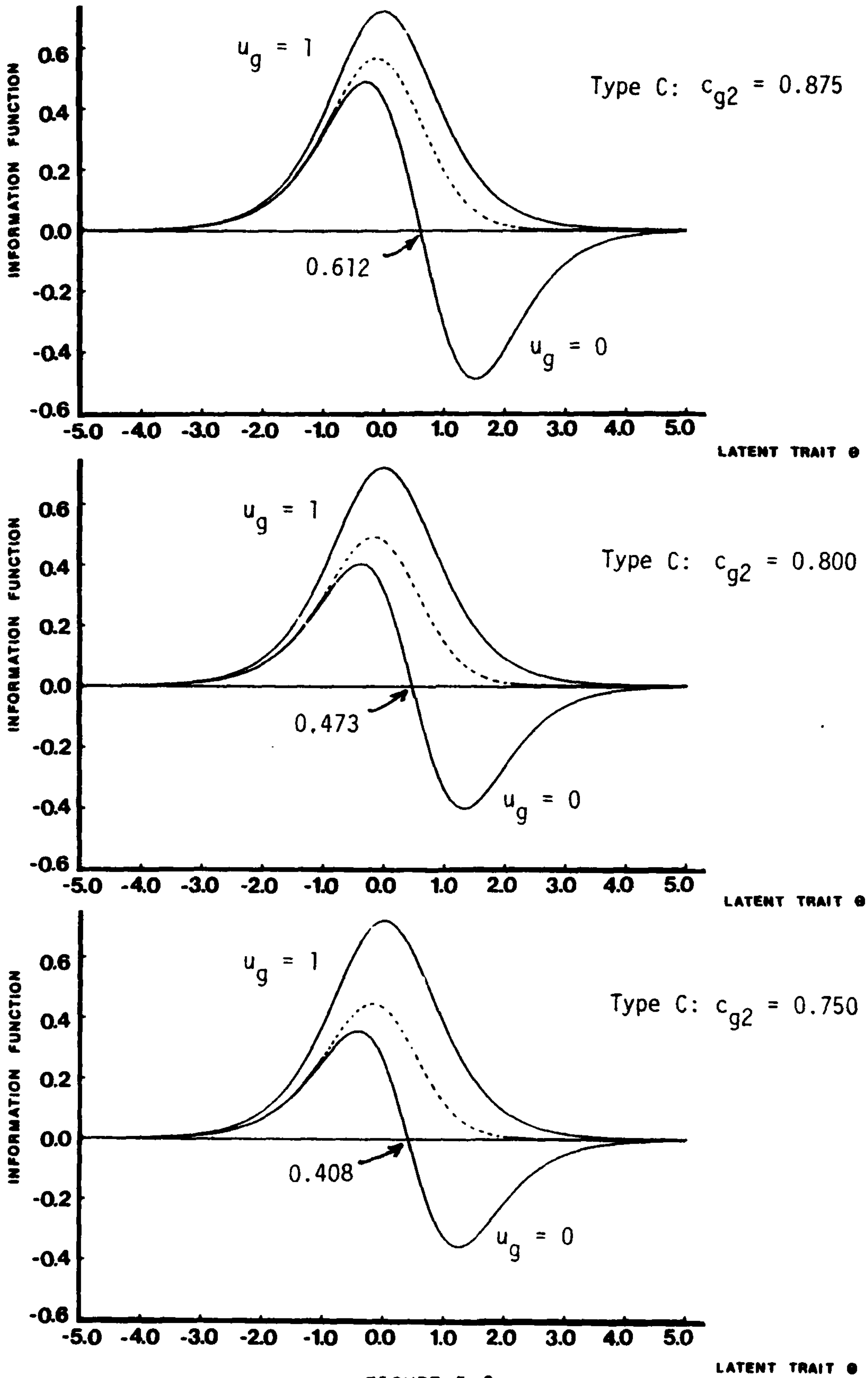


FIGURE 5-3

Item Response Information Function (Solid Lines) and Item Information Function (Dashed Line) of Type C Model: Logistic Function Is Used for $\Psi_g(\theta)$ with $a_g = 1.00$, $D = 1.7$ and $b_g = 0.00$, and specified c_{g2} .

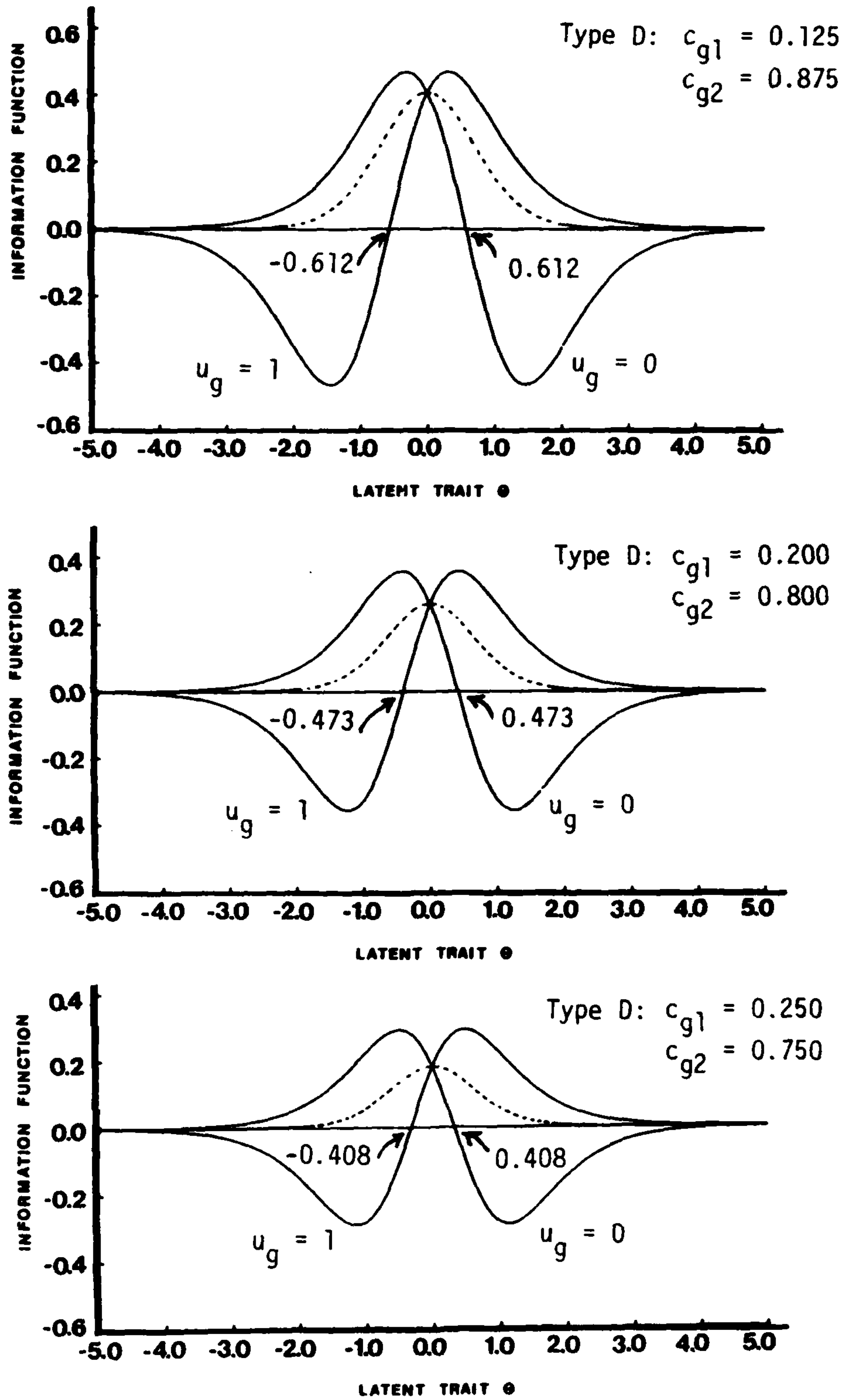


FIGURE 5-4

Item Response Information Function (Solid Lines) and Item Information Function (Dashed Line) of Type D Model: Logistic Function Is Used for $\Psi_g(\theta)$ with $a_g = 1.00$, $D = 1.7$ and $b_g = 0.00$, and c_{g1} and c_{g2} As Specified.

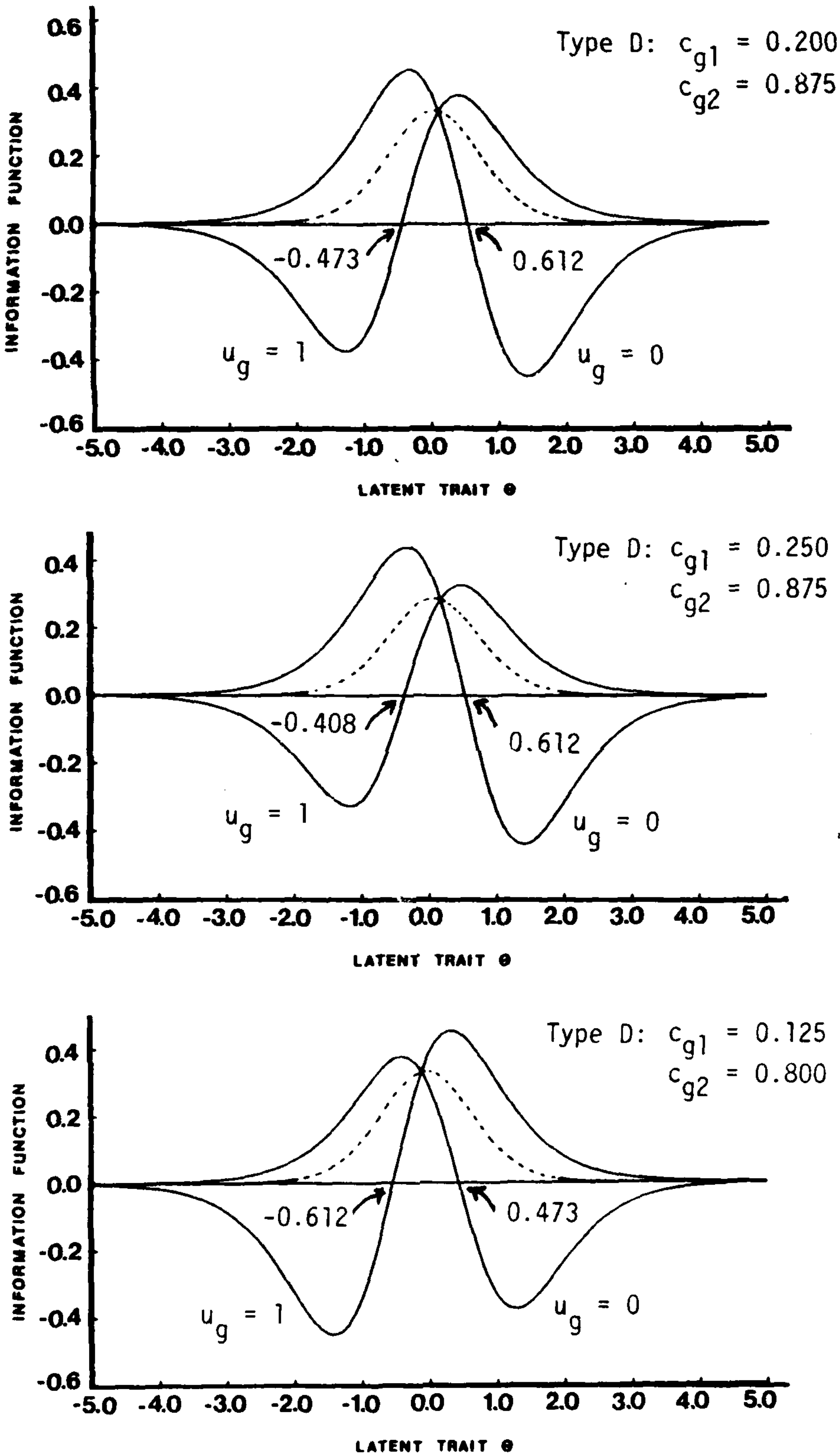
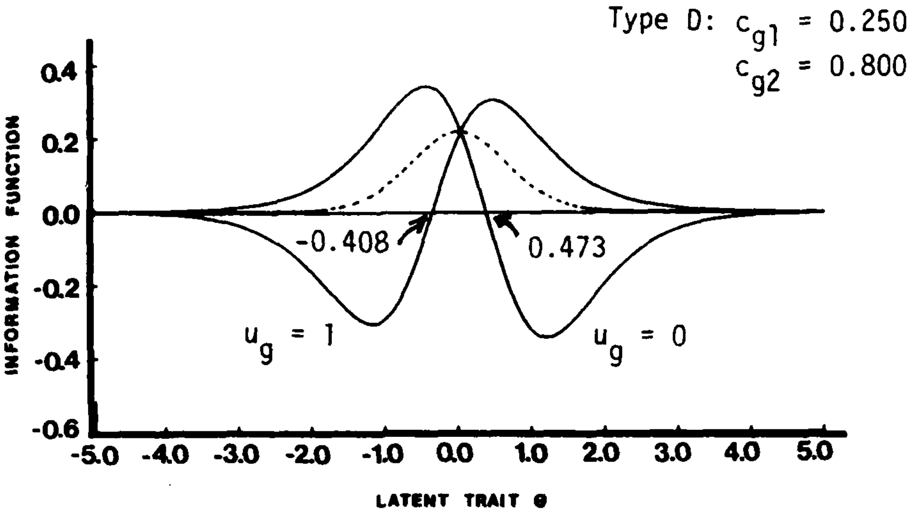
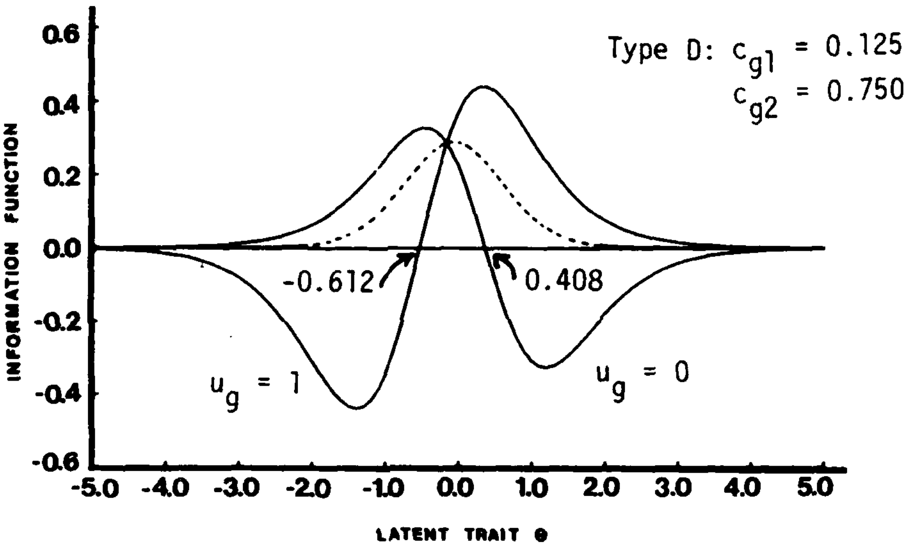


FIGURE 5-4 (Continued)





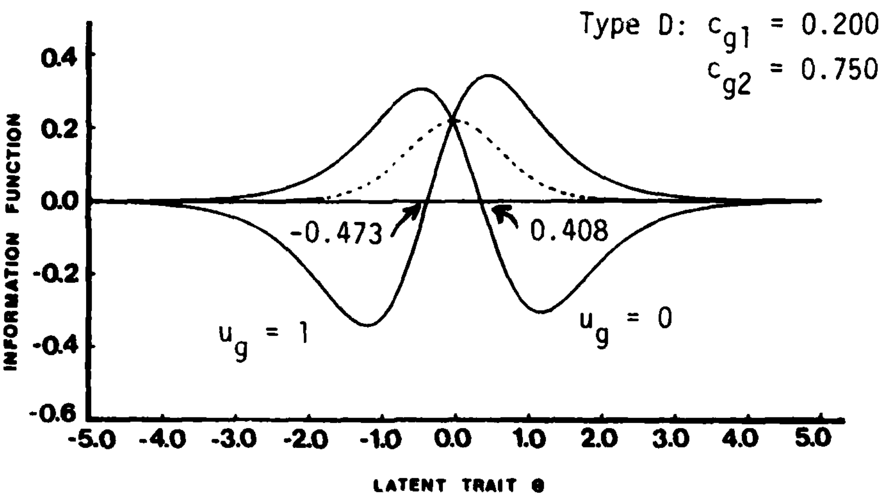


FIGURE 5-4 (Continued)

information function are not symmetric. We can see that, in each example, there is a subinterval on the negative side of θ for which the item response information function is negative for $u_g = 1$, and another subinterval on the positive side of θ for which it assumes negative values for $u_g = 0$.

Let $\underline{\theta}_g$ denote the critical value of θ below which the item response information function of Type B or D assumes negative values for $u_g = 1$, and $\bar{\theta}_g$ be the one above which the item response information function of Type C or D takes on negative values for $u_g = 0$. In general, we can write

$$(5.33) \qquad \underline{\theta}_g = -(2Da_g)^{-1} [\log c_{g2} - \log c_{g1}] + b_g$$

and

$$(5.34) \qquad \bar{\theta}_g = (2Da_g)^{-1} [\log (1-c_{g1}) - \log (1-c_{g2})] + b_g .$$

Since $b_g = 0.00$ in all our examples, the absolute values of $\underline{\theta}_g$ and $\bar{\theta}_g$ are the same whenever $c_{g1} = 1 - c_{g2}$. These critical values are shown in Figures 5-2 through 5-4.

Figure 5-5 presents the square root of the item information function for each of the three examples of Type B , in contrast to the one for Type A , which is drawn by a dotted line. We can see in this figure that, as c_{g1} becomes larger, the total information Q grows smaller. Since each example follows the three-parameter logistic model, the amount of information loss is the same as the one given in Table 4-1 , in which c_{g1} is given as c_g .

Similar results were obtained for the three examples of Type C ,

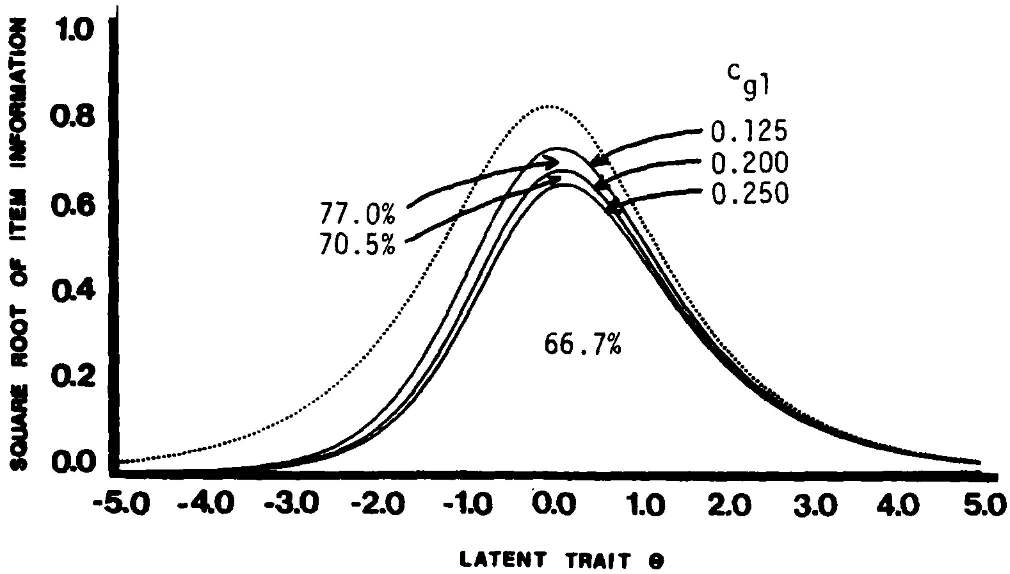


FIGURE 5-5

Square Root of the Item Information Function for Each of the Three Hypothetical Items of Type B (Solid Line) in Contrast to the One Following the Logistic Model with $D = 1.7$, $a_g = 1.00$ and $b_g = 0.00$ (Dotted Line). Values of c_{g1} Are: 0.125, 0.200 and 0.250, from Top to Bottom.

and are shown in Figure 5-6. Equation (3.18) indicates that the information loss in Type C is identical with the one in Type B whenever $c_{g2} = 1 - c_{g1}$. Thus the information loss for $c_{g2} = 0.875$ in Type C is the same as the one for $c_{g1} = 0.125$ in Type B, and so forth, and the amount of information loss for each of our three examples of Type C can be found in Table 4-1.

Equation (3.18) also indicates that the amount of information loss is additive for Type D, in which c_{g1} is non-zero and c_{g2} is not unity. Figure 5-7 presents the square root of the item information function for each of our nine examples of Type D, using four graphs for the sake of comparison. We can see in this figure that, in each case, the total information Q is substantially smaller than the one for Type A, which

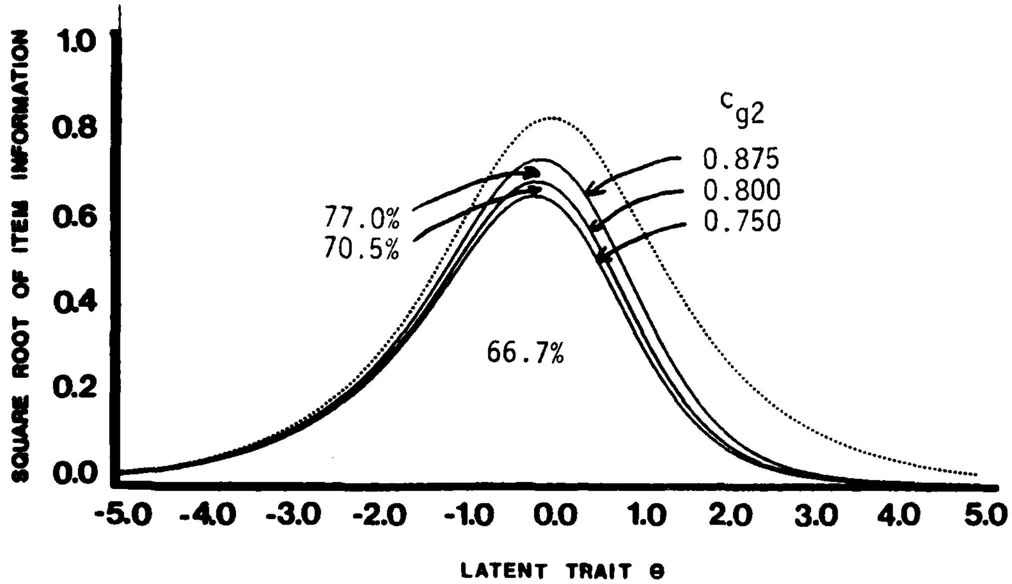


FIGURE 5-6

Square Root of the Item Information Function for Each of the Three Hypothetical Items of Type C (Solid Line) in Contrast to the One Following the Logistic Model with $D = 1.7$, $a_g = 1.00$ and $b_g = 0.00$ (Dotted Line). Values of c_{g2} Are: 0.875, 0.800 and 0.750, from Top to Bottom.

equals π. The amount of information loss in each case can be obtained, again, from Table 4-1, by simple additions. To give an example, if $c_{g1} = 0.125$ and $c_{g2} = 0.800$, then the amount of information loss is $0.723 + 0.927 = 1.650$, or $23.0\% + 29.5\% = 52.5\%$.

VI Speed of Convergence of the Conditional Distribution of the Maximum Likelihood Estimate, Given Latent Trait, to Normality

In a previous study, the speed of convergence of the conditional distribution of the maximum likelihood estimate $\hat{\theta}_V$, given θ, to normality was observed through a Monte Carlo study, using the constant information model, whose item characteristic function is given by (5.12) (cf. Samejima, RR-79-3). In that study, eight equally distant ability levels, which start from -3.0 and end with 2.6, were used, and one hundred examinees

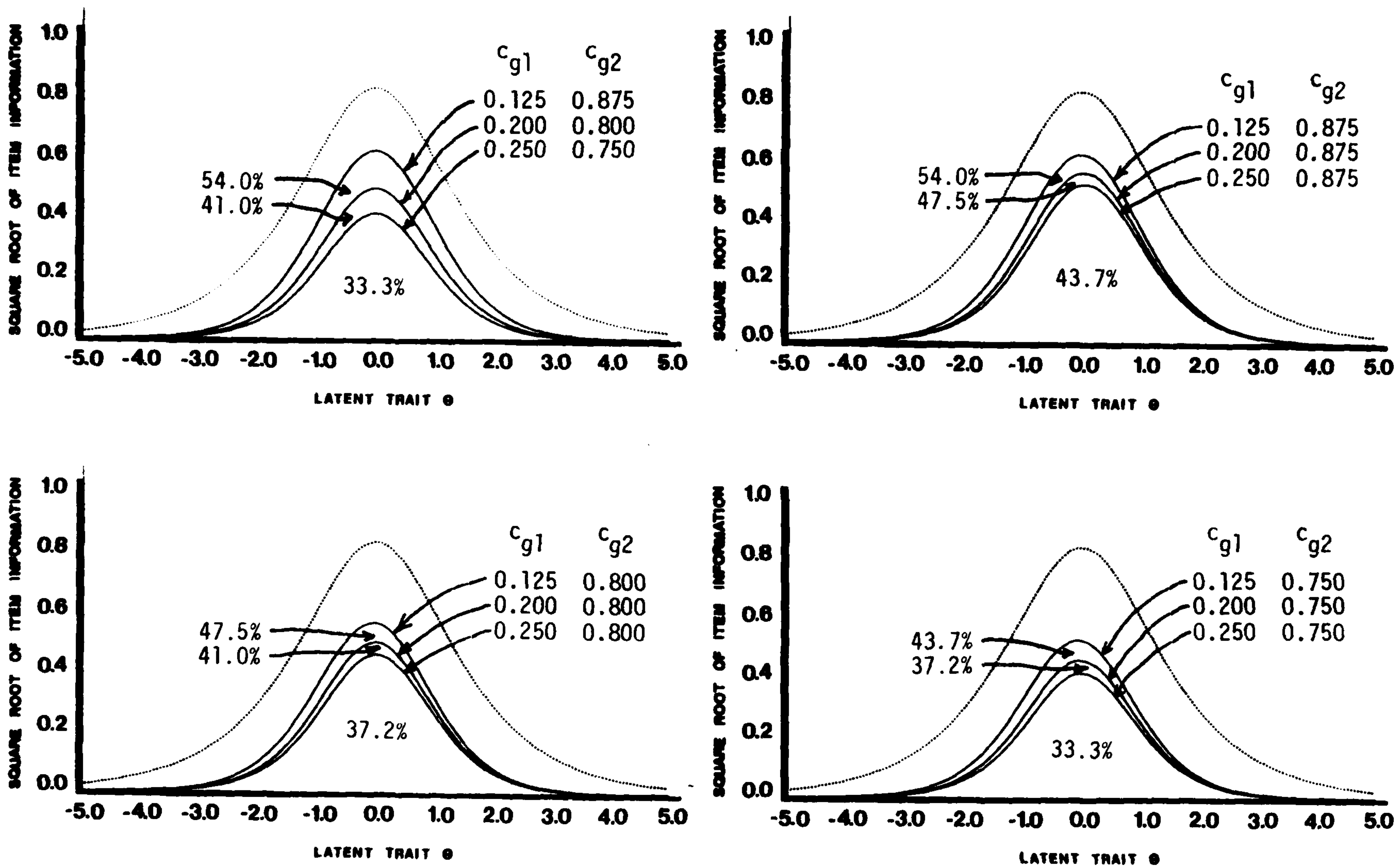


FIGURE 5-7

Square Root of the Item Information Function for Each of the Nine Hypothetical Items of Type D (Solid Line) Arranged for Comparison, in Contrast to the One in the Logistic Model with $D = 1.7$, $a_g = 1.00$ and $b_g = 0.00$ (Dotted Line). Values of c_{g1} and c_{g2} Are Specified in Each Graph.

were hypothesized on each ability level. There were twenty sessions in the hypothetical testing situation, and ten test items were administered in each session. Those items were all equivalent items, with the common parameters, $\gamma_g = 0.25$ and $\delta_g = 0.00$. The interval of θ for which each item characteristic function assumes positive values is, therefore, $-\pi < \theta < \pi$. The maximum likelihood estimate of ability was obtained for each hypothetical examinee, on the basis of the simple test score of all the test items administered up to and including the last session completed. If, for instance, session 17 was completed, then the maximum likelihood estimate of ability for each examinee would be obtained from his or her test score of the 170 test items. The resultant frequency distribution of the maximum likelihood estimates of the one hundred hypothetical examinees was compared with the asymptotic normal distribution, $N(\theta, I(\theta)^{-1/2})$.

For convenience, hereafter, we shall use τ instead of θ for the ability dimension in the above study. One rather crude measure of the speed of convergence to normality is the frequency of one of the terminal maximum values of the estimate. If this frequency is large, then the resultant frequency distribution will not be able to approximate the normal distribution. In the above example, these terminal maximum values are $-\pi$ and π .

Table 6-1 presents the frequencies of $-\pi$ on the left half, and those of π on the right half. To make the implication of the result easily understandable, the table was artificially enhanced, by using the frequencies of π for $\tau = 0.2, 1.0, 1.8$ and 2.6 , frequencies of $-\pi$ for $\tau = -0.2, -1.0, -1.8$ and -2.6 , frequencies of $-\pi$ for $\tau = -3.0, -2.2, -1.4$ and -0.6 , and frequencies of π for $\tau = 3.0, 2.2, 1.4$, and 0.6 , respectively. We can see from this table that the convergence to

TABLE 6-1

Freqencies of Lower (Left) and Upper (Right) Terminal Maximum Values, $-\pi$ and π, of the Maximum Likelihood Estimate of τ after Each Session, in Which Ten Equivalent Items in the Constant Information Model Were Given.

Session \ τ	-3.0	-2.6	-2.2	-1.8	-1.4	-1.0	-0.6	-0.2	0.2	0.6	1.0	1.4	1.8	2.2	2.6	3.0
1	100	75	62	37	13	8	3	0	0	3	8	13	37	62	75	100
2	100	66	35	11	1	1	0	0	0	0	1	1	11	35	66	100
3	100	52	20	5	1	0	0	0	0	0	0	1	5	20	52	100
4	100	43	13	2	0	0	0	0	0	0	0	0	2	13	43	100
5	99	39	4	1	0	0	0	0	0	0	0	0	1	4	39	99
6	95	30	2	0	0	0	0	0	0	0	0	0	0	2	30	95
7	94	26	0	0	0	0	0	0	0	0	0	0	0	0	26	94
8	94	23	0	0	0	0	0	0	0	0	0	0	0	0	23	94
9	94	20	0	0	0	0	0	0	0	0	0	0	0	0	20	94
10	93	15	0	0	0	0	0	0	0	0	0	0	0	0	15	93
11	91	14	0	0	0	0	0	0	0	0	0	0	0	0	14	91
12	90	12	0	0	0	0	0	0	0	0	0	0	0	0	12	90
13	89	10	0	0	0	0	0	0	0	0	0	0	0	0	10	89
14	87	9	0	0	0	0	0	0	0	0	0	0	0	0	9	87
15	84	6	0	0	0	0	0	0	0	0	0	0	0	0	6	84
16	80	6	0	0	0	0	0	0	0	0	0	0	0	0	6	80
17	79	4	0	0	0	0	0	0	0	0	0	0	0	0	4	79
18	78	1	0	0	0	0	0	0	0	0	0	0	0	0	1	78
19	76	1	0	0	0	0	0	0	0	0	0	0	0	0	1	76
20	73	0	0	0	0	0	0	0	0	0	0	0	0	0	0	73

normality becomes much slower as θ approaches either $-\pi$ or π.

We shall consider the transformation of τ to θ on which the common item characteristic function in the constant information model becomes the one in the model described by (2.4). This transformation is made by

$$(6.1) \qquad \theta = \Psi_g^{-1}[(c_{g2}-c_{g1})^{-1}\{\sin^2[\gamma_g(\tau-\delta_g)+(\pi/4)]-c_{g1}\}] .$$

We can see from (6.1) that the transformation depends upon the functional formula for $\Psi_g(\theta)$.

Equation (6.1) also implies that the lower asymptote of τ is the value of τ which satisfies

$$(6.2) \qquad \sin^2[\gamma_g(\tau-\delta_g)+(\pi/4)] - c_{g1} = 0 .$$

From (6.2) we obtain for the lower asymptote, $\underline{\underline{\tau}}$,

$$(6.3) \qquad \underline{\underline{\tau}} = \gamma_g^{-1}[\sin^{-1} c_{g1}^{1/2} - (\pi/4)] + \delta_g \ .$$

We also find from (6.1) that the upper asymptote of τ in the transformation is the value of τ which satisfies

$$(6.4) \qquad \sin^2[\gamma_g(\tau-\delta_g)+(\pi/4)] - c_{g1} = c_{g2}-c_{g1} \ .$$

Thus we can write from (6.4) for the upper asymptote, $\bar{\bar{\tau}}$,

$$(6.5) \qquad \bar{\bar{\tau}} = \gamma_g^{-1}[\sin^{-1} c_{g2}^{1/2} - (\pi/4)] + \delta_g \ .$$

Note that neither $\underline{\underline{\tau}}$ nor $\bar{\bar{\tau}}$ is affected by the functional formula for $\Psi_g(\theta)$, and, except for the parameters γ_g and δ_g , $\underline{\underline{\tau}}$ depends solely upon c_{g1} , and $\bar{\bar{\tau}}$ upon c_{g2} . (6.3) and (6.5) imply that these two asymptotes equal the lower and upper asymptotes of the interval of τ for which the item characteristic function in the constant information model assumes positive values, if, and only if, $c_{g1} = 0.000$ and $c_{g2} = 1.000$. Note that, since models of Type A satisfy this condition, Table 6-1 is directly applicable for any model of Type A , if we transform τ to θ through (6.1) with $c_{g1} = 0.000$ and $c_{g2} = 1.000$ and specifying Ψ^{-1} . For the other three types, either all the maximum likelihood estimates which are less than, or equal to, $\underline{\underline{\tau}}$ must be transformed to the lower terminal maximum value of θ , or those which are greater than, or equal to, $\bar{\bar{\tau}}$ to the upper terminal maximum value of θ , or both.

Since in our example $\gamma_g = 0.25$ and $\delta_g = 0.00$, for $c_{g1} = 0.200$ we obtain $\underline{\underline{\tau}} = -1.287$, and for $c_{g2} = 0.800$ we have $\bar{\bar{\tau}} = 1.287$. Table 6-2 presents the frequencies of the lower terminal maximum thus increased by setting $\underline{\underline{\tau}} = -1.287$, as well as those of the upper terminal by setting $\bar{\bar{\tau}} = 1.287$. For convenience, the values of τ are still used in the table instead of the transformed values of θ . Since, by transformation, the first five values of τ in the upper half of the table and the last five values of τ in the lower half of the table are thrown out of the range, the frequencies in those ten columns would have been unobservable, had we used the Monte Carlo method directly by adopting the types of models we are considering now. If our model is of Type D with $c_{g1} = 0.200$ and $c_{g2} = 0.800$, then those frequencies in Table 6-2 excluding the above ten columns should be considered. If our model is of Type C with $c_{g2} = 0.800$, then the lower half of Table 6-2 , excluding the last five columns, plus the left half of Table 6-1 should be adopted for our consideration. If our model is of Type B with $c_{g1} = 0.200$, then the upper half of Table 6-2 , excluding the first five columns, plus the right half of Table 6-1 should be considered.

When $\Psi_g(\theta)$ is specified by the logistic function shown by (5.22), we can rewrite (6.1) into the form

$$(6.6) \qquad \theta = (Da_g)^{-1}[\log\{\sin^2[\gamma_g(\tau-\delta_g)+(\pi/4)]-c_{g1}\} \\ -\log\{c_{g2}-\sin^2[\gamma_g(\tau-\delta_g)+(\pi/4)]\}] + b_g \ .$$

If we set $\gamma_g = 0.25$ and $\delta_g = 0.00$, as we did before, then (6.6) will depend solely upon the two, three or four parameters of the model we choose. If, further, we set $b_g = 0.00$, and $c_{g1} = 0.200$ for models of Types B and D and $c_{g2} = 0.800$ for models of Types C and D , then

TABLE 6-2

Frequencies of Lower Terminal Maximum Value of the Maximum Likelihood Estimate of θ (Above) and Those of the Upper Terminal Maximum Value of the Maximum Likelihood Estimate of θ (Below) When θ Is Transformed from τ with $c_{g1} = 0.200$ and $c_{g2} = 0.800$, Respectively, after the Completion of Each of the Twenty Sessions. Table Was Converted from Table 6-1 .

Session \ τ	-3.0	-2.6	-2.2	-1.8	-1.4	-1.0	-0.6	-0.2	0.2	0.6	1.0	1.4	1.8	2.2	2.6	3.0
1	100	100	99	89	65	48	27	12	4	1	0	0	0	0	0	0
2	100	100	100	90	73	40	10	1	0	0	0	0	0	0	0	0
3	100	100	100	93	68	31	2	0	0	0	0	0	0	0	0	0
4	100	100	100	97	74	26	2	0	0	0	0	0	0	0	0	0
5	100	100	100	96	79	22	1	0	0	0	0	0	0	0	0	0
6	100	100	100	97	73	21	0	0	0	0	0	0	0	0	0	0
7	100	100	100	98	78	17	0	0	0	0	0	0	0	0	0	0
8	100	100	100	99	78	13	0	0	0	0	0	0	0	0	0	0
9	100	100	100	98	75	12	0	0	0	0	0	0	0	0	0	0
10	100	100	100	99	76	8	0	0	0	0	0	0	0	0	0	0
11	100	100	100	99	75	7	0	0	0	0	0	0	0	0	0	0
12	100	100	100	100	80	7	0	0	0	0	0	0	0	0	0	0
13	100	100	100	100	79	7	0	0	0	0	0	0	0	0	0	0
14	100	100	100	100	80	7	0	0	0	0	0	0	0	0	0	0
15	100	100	100	100	81	5	0	0	0	0	0	0	0	0	0	0
16	100	100	100	100	84	5	0	0	0	0	0	0	0	0	0	0
17	100	100	100	100	81	3	0	0	0	0	0	0	0	0	0	0
18	100	100	100	100	80	3	0	0	0	0	0	0	0	0	0	0
19	100	100	100	100	78	3	0	0	0	0	0	0	0	0	0	0
20	100	100	100	100	79	3	0	0	0	0	0	0	0	0	0	0

Session \ τ	-3.0	-2.6	-2.2	-1.8	-1.4	-1.0	-0.6	-0.2	0.2	0.6	1.0	1.4	1.8	2.2	2.6	3.0
1	0	0	0	0	0	0	1	4	12	27	48	65	89	99	100	100
2	0	0	0	0	0	0	0	0	1	10	40	73	90	100	100	100
3	0	0	0	0	0	0	0	0	0	2	31	68	93	100	100	100
4	0	0	0	0	0	0	0	0	0	2	26	74	97	100	100	100
5	0	0	0	0	0	0	0	0	0	1	22	79	96	100	100	100
6	0	0	0	0	0	0	0	0	0	0	21	73	97	100	100	100
7	0	0	0	0	0	0	0	0	0	0	17	78	98	100	100	100
8	0	0	0	0	0	0	0	0	0	0	13	78	99	100	100	100
9	0	0	0	0	0	0	0	0	0	0	12	75	98	100	100	100
10	0	0	0	0	0	0	0	0	0	0	8	76	99	100	100	100
11	0	0	0	0	0	0	0	0	0	0	7	75	99	100	100	100
12	0	0	0	0	0	0	0	0	0	0	7	80	100	100	100	100
13	0	0	0	0	0	0	0	0	0	0	7	79	100	100	100	100
14	0	0	0	0	0	0	0	0	0	0	7	80	100	100	100	100
15	0	0	0	0	0	0	0	0	0	0	5	81	100	100	100	100
16	0	0	0	0	0	0	0	0	0	0	5	84	100	100	100	100
17	0	0	0	0	0	0	0	0	0	0	3	81	100	100	100	100
18	0	0	0	0	0	0	0	0	0	0	3	80	100	100	100	100
19	0	0	0	0	0	0	0	0	0	0	3	78	100	100	100	100
20	0	0	0	0	0	0	0	0	0	0	3	79	100	100	100	100

(6.6) will depend soley upon the discrimination parameter, a_g .

Since we can write for the slope of the item characteristic function, $P^*_g(\tau)$, in the constant information model

$$(6.7) \qquad \frac{\partial}{\partial\tau} P^*_g(\tau) = 2\gamma_g \sin[\gamma_g(\tau-\delta_g)+(\pi/4)] \cos[\gamma_g(\tau-\delta_g)+(\pi/4)] ,$$

with $\gamma_g = 0.25$ and $\delta_g = 0.00$, we obtain $\frac{\partial}{\partial\tau} P^*_g(0) = 0.25$. For the corresponding slope of the item characteristic function, $P_g(\theta)$, we obtain

$$(6.8) \qquad \frac{\partial}{\partial\theta} P_g(\theta) = (c_{g2}-c_{g1})Da_g\Psi_g(\theta)[1-\Psi_g(\theta)] ,$$

and with $b_g = 0.00$ we have $\frac{\partial}{\partial\theta} P_g(0) = 0.25(c_{g2}-c_{g1})Da_g$. If we equate these two slopes, we obtain for the discrimination parameter

$$(6.9) \qquad a_g = D^{-1}(c_{g2}-c_{g1})^{-1} .$$

When we set $D = 1.7$, and $c_{g1} = 0.200$ for Types B and D and $c_{g2} = 0.800$ for Types C and D , we have for Type A $a_g \doteq 0.58824$, for Types B and C $a_g \doteq 0.73529$, and for Type D $a_g \doteq 0.98039$. Thus we can see that, in order to maintain the same slope at the origin of the scale as the original common item characteristic function in the constant information model, we must choose the discrimination parameter specified above, for each type of model. Because of the noise, the discrimination parameter must be larger for the model of Type B or C in comparison with the one for the logistic model, which is of Type A , and it must be largest for the model of Type D .

A similar result is obtained in this specific example when we equate the amount of item information at the origin of the scale. Since we have for the item information function $I^*_g(\tau)$

$$(6.10) \qquad I^*_g(\tau) = [\frac{\partial}{\partial\tau} P^*_g(\tau)]^2[P^*_g(\tau)\{1-P^*_g(\tau)\}]^{-1} ,$$

we obtain for the above parameters of the common item characteristic function in the constant information model $I^*_g(0) = 0.25$. On the other hand, from (1.7), (1.2), (2.4) and (6.8) we can write

$$(6.11) \qquad I_g(\theta) = (c_{g2}-c_{g1})^2D^2a_g{}^2\{\Psi_g(\theta)\}^2[1-\Psi_g(\theta)]^2 [c_{g1}+(c_{g2}-c_{g1})\Psi_g(\theta)]^{-1}[(1-c_{g1})-(c_{g2}-c_{g1})\Psi_g(\theta)]^{-1} .$$

From (6.11), we obtain $I_g(0) = 0.25(c_{g2}-c_{g1})^2D^2a_g{}^2(c_{g1}+c_{g2})^{-1}(2-c_{g1}-c_{g2})^{-1}$. Equating these two amounts of item information, we can write for the discrimination parameter a_g

$$(6.12) \qquad a_g = D^{-1}(c_{g1}+c_{g2})^{1/2}(2-c_{g1}-c_{g2})^{1/2}(c_{g2}-c_{g1})^{-1} .$$

With $D= 1.7$ and $c_{g1} = 0.200$ for Types B and D and $c_{g2} = 0.800$ for Types C and D , we obtain $a_g \doteq 0.58824$ for Type A , $a_g \doteq 0.72044$ for Types B and C , and $a_g \doteq 0.98039$ for Type D .

Tables 6-3 through 6-6 present the values of θ transformed by (6.6) for each of the four types of models, with $\gamma_g = 0.25$, $\delta_g = 0.00$, $b_g = 0.00$, $D = 1.7$, and $c_{g1} = 0.200$ for Types B and D and $c_{g2} = 0.800$ for Types C and D , using four different values of a_g ,

TABLE 6-3

Transformed θ from τ with the Logistic Function with $D = 1.7$, $a_g = 0.50$ and $b_g = 0.00$ for $\Psi_g(\theta)$. $c_{g1} = 0.200$ for Types B and D , $c_{g2} = 0.800$ for Types C and D . Parameters fur the Original Item Characteristic Function in the Constant Information Model Are $\gamma_g = 0.25$ and $\delta_g = 0.00$.

Type / τ	A	B	C	D
-3.0	-7.860	---	-7.598	---
-2.6	-4.690	---	-4.422	---
-2.2	-3.359	---	-3.080	---
-1.8	-2.480	---	-2.181	---
-1.4	-1.801	---	-1.473	---
-1.0	-1.229	-2.950	-0.858	-2.579
-0.6	-0.717	-1.704	-0.282	-1.269
-0.2	-0.236	-0.927	0.296	-0.395
0.2	0.236	-0.296	0.927	0.395
0.6	0.717	0.282	1.704	1.269
1.0	1.229	0.858	2.950	2.579
1.4	1.801	1.473	---	---
1.8	2.480	2.181	---	---
2.2	3.359	3.080	---	---
2.6	4.690	4.422	---	---
3.0	7.860	7.598	---	---

TABLE 6-4

Transformed θ from τ with the Logistic Function with $D = 1.7$, $a_g = 1.00$ and $b_g = 0.00$ for $\Psi_g(\theta)$. $c_{g1} = 0.200$ for Types B and D, $c_{g2} = 0.800$ for Types C and D . Parameters for the Original Item Characteristic Function in the Constant Information Model Are $\gamma_g = 0.25$ and $\delta_g = 0.00$.

Type / τ	A	B	C	D
-3.0	-3.930	---	-3.799	---
-2.6	-2.345	---	-2.211	---
-2.2	-1.680	---	-1.540	---
-1.8	-1.240	---	-1.091	---
-1.4	-0.900	---	-0.736	---
-1.0	-0.614	-1.475	-0.429	-1.289
-0.6	-0.358	-0.852	-0.141	-0.635
-0.2	-0.118	-0.464	0.148	-0.198
0.2	0.118	-0.148	0.464	0.198
0.6	0.358	0.141	0.852	0.635
1.0	0.614	0.429	1.475	1.289
1.4	0.900	0.736	---	---
1.8	1.240	1.091	---	---
2.2	1.680	1.540	---	---
2.6	2.345	2.211	---	---
3.0	3.930	3.799	---	---

TABLE 6-5

Transformed θ from τ with the Logistic Function with $D = 1.7$, $a_g = 1.50$ and $b_g = 0.00$ for $\Psi_g(\theta)$. $c_{g1} = 0.200$ for Types B and D, $c_{g2} = 0.800$ for Types C and D. Parameters for the Original Item Characteristic Function in the Constant Information Model Are $\gamma_g = 0.25$ and $\delta_g = 0.00$.

Type / τ	A	B	C	D
-3.0	-2.620	---	-2.533	---
-2.6	-1.563	---	-1.474	---
-2.2	-1.120	---	-1.027	---
-1.8	-0.827	---	-0.727	---
-1.4	-0.600	---	-0.491	---
-1.0	-0.410	-0.983	-0.286	-0.860
-0.6	-0.239	-0.568	-0.094	-0.423
-0.2	-0.079	-0.309	0.099	-0.132
0.2	0.079	-0.099	0.309	0.132
0.6	0.239	0.094	0.568	0.423
1.0	0.410	0.286	0.983	0.860
1.4	0.600	0.491	---	---
1.8	0.827	0.727	---	---
2.2	1.120	1.027	---	---
2.6	1.563	1.474	---	---
3.0	2.620	2.533	---	---

TABLE 6-6

Transformed θ from τ with the Logistic Function with $D = 1.7$, $a_g = 2.0$ and $b_g = 0.00$ for $\Psi_g(\theta)$. $c_{g1} = 0.200$ for Types B and D, $c_{g2} = 0.800$ for Types C and D. Parameters for the Original Item Characteristic Function in the Constant Information Model Are $\gamma_g = 0.25$ and $\delta_g = 0.00$.

Type / τ	A	B	C	D
-3.0	-1.965	---	-1.899	---
-2.6	-1.173	---	-1.106	---
-2.2	-0.840	---	-0.770	---
-1.8	-0.620	---	-0.545	---
-1.4	-0.450	---	-0.368	---
-1.0	-0.307	-0.737	-0.214	-0.645
-0.6	-0.179	-0.426	-0.071	-0.317
-0.2	-0.059	-0.232	0.074	-0.099
0.2	0.059	-0.074	0.232	0.099
0.6	0.179	0.071	0.426	0.317
1.0	0.307	0.214	0.737	0.645
1.4	0.450	0.368	---	---
1.8	0.620	0.545	---	---
2.2	0.840	0.770	---	---
2.6	1.173	1.106	---	---
3.0	1.965	1.899	---	---

0.50 , 1.00 , 1.50 and 2.00 , respectively, for each of the sixteen values of τ shown in Tables 6-1 and 6-2 . Comparison of the result for Type B with that for Type A reveals that the values of θ corresponding to $\tau = -2.2$ for Type A are a little larger in absolute value than those corresponding to $\tau = -1.0$. If we compare the frequencies in the column for $\tau = -1.0$ in the upper half of Table 6-2 in comparison with those in the column for $\tau = -2.2$ in the left half of Table 6-1 , however, we find that the convergence is substantially slower in the former case. Although in the former no frequencies of the lower terminal maximum are observed after administering seventy equivalent items, in the latter we still have three after administering two hundred items, which shows the effect of noise in the models of Type B on the speed of convergence of the conditional distribution of the maximum likelihood estimate, given θ . If we compare the columns for Type B across the four tables. we will find out this slow convergence occurs at the level of θ closer to the origin as the discrimination index, a_g , increases. The values of θ corresponding to $\tau = -1.0$ are: -2.950 , -1.475 , -0.983 and -0.737 for $a_g = 0.5$, 1.0 , 1.5 and 2.0 , respectively, the fact which indicates that the effect of noise is much more serious when the discrimination parameter a_g is greater. An identical observation can be made when we compare the result for Type C with the one for Type A , in which the frequencies of the upper terminal maximum are considered. The effect of noise is even more conspicuous for Type D , as is expected from its formula for the item characteristic function. These four tables disclose that the value of θ for $\tau = -1.0$ for Type D is approximately the same as the value of θ for $\tau = -1.8$ for Type A in each case, and comparison of the frequencies in the column for

$\tau = -1.0$ in the upper half of Table 6-2 with those in the column for $\tau = 1.8$ in the left half of Table 6-1 reveals a greater contrast. In the latter, the convergence is substantially faster, showing no frequencies of the lower terminal maximum after the administration of sixty equivalent items. An identical result is observed for the frequencies of the upper terminal maximum. It should be noted that these slow convergences are observed at $\theta = -0.645$ and $\theta = 0.645$, i.e., close to the origin, for $a_g = 2.0$, which indicates even more serious effect of the noise in Type D models when the discrimination parameter is large.

VII <u>Discussion and Conclusions</u>

The effect of noise in models for the dichotomous item has been observed, with respect to three types of models, Types B , C and D , in comparison with Type A , which does not include noise. It has been shown that the information loss caused by the noise is substantial, and the speed of convergence of the conditional distribution of the maximum likelihood estimate of the latent trait, given the true latent trait, to normality decreases substantially.

The author's standpoint has been that we should try to eliminate noise by constructing "good" test items, since noise, which may be caused by guessing, is nothing but nuisance, and its undesirable effect is probably greater than most researchers think. This attitude of the author is reflected in the paper published as early as in 1972 (Samejima, 1972), in which a general model for free-response data is proposed. It is also reflected in the proposal of a new family of models (Samejima, RR-79-4), which implies that "good distractors" in the multiple-choice test item

will effectively reduce noise. If we do not have a technique to extract precious metal from ore, ore itself will be useless, however precious the contained metal may be. In the three-parameter logistic model, for example, there is no such technique involved, and the item characteristic function is defined for the "correct" answer, which is contaminated by noise.

Because of general indifference and the wide acceptance of the three-parameter logistic model, however, it seems necessary that someone should quantify the effect of noise incorporated in such models, and clarify what we sould do to supplement the loss, if possible. The present paper is the first of such attempts. If, for example, some researchers wish to use the author's methods and approaches for estimating the operating characteristics of discrete item responses (Samejima, 1977, RR-77-1, RR-78-1 through RR-78-6, RR-80-2, RR-80-4, RR-81-2, RR-81-3, Final Report), using a set of test items following the three-parameter logistic model as the Old Test, they should take the effect of noise into consideration; without it the research may even by ruined.

In the future, this problem of noise may not be a serious consideration. With the progress of technology and a wider use of on-line systems, the multiple-choice test items will become less and less important, and with a proper effort by researchers free-response test items will become more and more common. In such a case models for free-response data, including both for discrete test items and for continuous test items (Samejima, 1973a, 1974) will become more useful. In the meantime, however, we must be extra careful in handling models which include noise, like the three-parameter logistic model.

REFERENCES

[1] Birnbaum, A. Some latent trait models and their use in inferring an examinee's ability. In F. M. Lord and M. R. Novick; Statistical theories of mental test scores. Addison-Wesley, 1968, Chapters 17-20.

[2] Lazarsfeld, P. F. Latent structure analysis. In S. Koch (Ed.), Psychology: a study of a science, Volume 3. McGraw-Hill, 1959, pages 476-542.

[3] Lord, F. M. and M. R. Novick. Statistical theories of mental test scores. Addison-Wesley, 1968.

[4] Samejima, F. Estimation of latent ability using a response pattern of graded scores. Psychometrika Monograph, No. 17, 1969.

[5] Samejima, F. A general model for free-response data. Psychometrika Monograph, No. 18, 1972.

[6] Samejima, F. Homogeneous case of the continuous response model. Psychometrika, 38, 1973a, pages 203-219.

[7] Samejima, F. A comment on Birnbaum's three-parameter logistic model in the latent trait theory. Psychometrika, 38, 1973b, pages 221-233.

[8] Samejima, F. Normal ogive model on the continuous response level in the multidimensional latent space. Psychometrika, 39, 1974, pages 111-121.

[9] Samejima, F. A method of estimating item characteristic functions using the maximum likelihood estimate of ability. Psychometrika, 42, 1977, pages 163-191.

ONR TECHNICAL REPORTS AND OTHER PAPERS UNDER CONTRACT NO. N00014-77-C-0360, 1977-81

(1) Samejima, F. Estimation of the operating characteristics of item response categories I: Introduction to the Two-Parameter Beta Method. RR-77-1, 1977.

(2) Samejima, F. The application of graded response models. Proceedings of the 1977 Computerized Adaptive Testing Conference (D.J. Weiss, Ed.), 28-37, 1977.

(3) Samejima, F. Future directions for computerized adaptive testing (panel discussion). Proceedings of the 1977 Computerized Adaptive Testing Conference (D.J. Weiss, Ed.), 430-440, 1977.

(4) Samejima, F. Estimation of the operating characteristics of item response categories II: Further development of the Two-Parameter Beta Method. RR-78-1, 1978.

(5) Samejima, F. Estimation of the operating characteristics of item response categories III: The Normal Approach Method and the Pearson System Method. RR-78-2, 1978.

(6) Samejima, F. Estimation of the operating characteristics of item response categories IV: Comparison of the different methods. RR-78-3, 1978.

(7) Samejima, F. Estimation of the operating characteristics of item response categories V: Weighted Sum Procedure in the Conditional P.D.F. Approach. RR-78-4, 1978.

(8) Samejima, F. Estimation of the operating characteristics of item response categories VI: Proportioned Sum Procedure in the Conditional P.D.F. Approach. RR-78-5, 1978.

(9) Samejima, F. Estimation of the operating characteristics of item response categories VII: Bivariate P.D.F. Approach with Normal Approach Method. RR-78-6, 1978.

(10) Samejima, F. Constant Information Model: A new, promising item characteristic function. RR-79-1, 1979.

(11) Samejima, F. and P. S. Livingston. Method of moments as the least squares solution for fitting a polynomial. RR-79-2, 1979.

(12) Samejima, F. Convergence of the conditional distribution of the maximum likelihood estimate, given latent trait, to the asymptotic normality: Observations made through the Constant Information Model. RR-79-3, 1979.

(13) Samejima, F. A new family of models for the multiple-choice item. RR-79-4, 1979.

(14) Samejima, F. Constant Information Model on the dichotomous response level. Proceedings of the 1979 Computerized Adaptive Testing Conference (D.J. Weiss, Ed.), 145-163, 1979.

(15) Samejima, F. Research on the multiple-choice test item in Japan: Toward the validation of mathematical models. ONR-Tokyo, Scientific Monograph 3, April, 1980.

(16) Samejima, F. & R. L. Trestman. Analysis of Iowa data I: Initial study and findings. RR-80-1, 1980.

(17) Samejima, F. Estimation of the operating characteristics when the test information of the Old Test is not constant I: Rationale. RR-80-2, 1980.

(18) Samejima, F. Is Bayesian estimation proper for estimating the individual's ability? RR-80-3, 1980.

(19) Samejima, F. Estimation of the operating characteristics when the test information of the Old Test is not constant II: Simple Sum Procedure of the Conditional P.D.F. Approach/Normal Approach Method using three subtests of the Old Test. RR-80-4, 1980.

(20) Samejima, F. An alternative estimator for the maximum likelihood estimator for the two extreme response patterns. RR-81-1, 1981.

(21) Samejima, F. Estimation of the operating characteristics when the test information of the Old Test is not constant II: Simple Sum Procedure of the Conditional P.D.F. Approach/Normal Approach Method using three subtests of the Old Test. No. 2. RR-81-2, 1981.

(22) Samejima, F. & P. S. Changas. How small the number of the test items can be for the basis of estimating the operating characteristics of the discrete responses to unknown test items. RR-81-3, 1981.

(23) Samejima, F. Final Report: Efficient methods of estimating the operating characteristics of item response categories and challenge to a new model for the multiple-choice item. Final Report, 1981.

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